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COMPUTER-AIDED-DETECTION VALIDATION STUDY.(U)

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FINAL REPORT OF COMPUTER-AIDED-DETECTION
VALIDATION STUDY,

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ABSTRACT

The performance of a computer-aided detection model has been validated using a large set of recorded sea test data. The data base, processing procedures and results obtained are described.

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1. INTRODUCTION

The classical approach to sonar target detection has been to provide a sonar operator with audio and visual displays of the information received by the sonar system. This approach is known to have the following deficiencies:

(a) The data rates associated with a modern sonar system often exceed the capacity provided by state-of-the-art display technology and/or the capacity of the operator to process the information displayed.

(b) Sonar operators do not perform in an optimal way when faced with a rare event situation.

(c) It is very difficult to obtain consistent application of the same detection criteria when several sonar operators are used.

The deficiencies listed above have motivated the investigation of the use of a computer to aid in the detection process. Previous investigation of a computer-aided detection (CAD) model, developed under Contract NObsr-93352, indicates that a CAD model will yield unalerted detection performance approximately equal to that of an alerted operator. These results were based largely on computer generated data.

The aim of the work discussed in this report is to achieve the next logical step, that of validating the CAD model performance by the application of the model to a large volume of recorded sea test data. The sea test data base used and the data reduction performed are described in Section 2. The results obtained by processing approximately 1200 ping cycles of sea data are described in Section 3. The internal logic of the CAD

model is described in Appendix A. Appendix B provides a derivation of the transformation from peak height to log likelihood ratio. A simplified model for predicting CAD performance is described in Appendix C. Appendix D provides several sets of displays obtained with sea data to give a visual presentation of the effect of the CAD model on a display.

The results obtained in this study indicate that the CAD model can detect submarine target tracks with a signal-to-noise ratio at the output of the signal processor as low as 11 dB. This performance compares favorably with that obtained by a sonar operator. The data rate required to display the output of the CAD model is substantially reduced relative to that required to directly display the output of the signal processor.

2. DESCRIPTION OF DATA PROCESSING PROCEDURES

(U) The data processing procedures implemented to accomplish the validation of the computer-aided detection (CAD) model are described in the following sections. This chapter concentrates on the procedures applied external to the basic CAD model. The internal data processing procedures of the CAD model are described in detail in Appendix A.

(U) To provide a starting point for the description of the data processing procedures, the sea test data base used is described in Section 2.1. Section 2.2 describes the functional data flow within the processing procedures, and is organized to parallel the order in which basic computer runs were executed to accomplish the processing procedures.

2.1 DATA BASE DESCRIPTION

(C) Data gathered during the technical evaluation of the AN/SQS-26 sonar was used as the data base for the validation of the CAD model. The TECHEVAL data included about 4,000 ping cycles divided into about 200 runs of about 20 ping cycles per run. From this data base about 1250 ping cycles were selected and processed through the CAD model. The output of the shipboard signal processing unit (and other information) was recorded on 1 inch 14 channel instrumentation tape by an analog data acquisition system (ADAS). The recorded analog information was converted to digital format as part of the data processing accomplished in support of TECHEVAL. The digitized data was formatted to have one digital file for each ping cycle. Within each digital file, the data consisted of samples taken during two time gates. The first time gate occurred during transmit and was approximately ½ second long. The second time gate

occurred during receive, and was approximately 12 seconds long. The position of the second time gate was controlled by the sonar operator's cursor. In this way the 12-second interval always overlapped the target signal and the transponder signal.

(U) During each time gate data was sampled at a 1 kHz rate from nine analog channels. A time code channel, recorded on the analog tape, also was decoded and stored in the digital file at a 1 kHz rate. The nine analog channels which were sampled were:

(C) (1) SSI

(2) CW AGC IN

(3) CW IN

(4) CW OUT

(5) CP AGC IN

(6) CP IN

(7) CP OUT

(8) 3-BIT CP OUT

(9) DELTIC REF.

(U) For the program to validate the CAD model, the only information that was needed from the digital files was:

(U) (1) Time Codes. By comparing the time codes in the first gate with those in the second time gate the time of signal arrival relative to the time of transmit was determined.

(U) (2) CP OUT. The output of the CODED PULSE processor was used as the basic input to the CAD model.

(C) In addition to the sonar data discussed above, the sonar operator recorded a response to the signal received on each ping. The responses were: NO SIGNAL, WEAK, MEDIUM, and STRONG. These operator responses were punched on cards for use in the single ping comparison of CAD performance with operator responses.

2.2 FUNCTIONAL DATA FLOW

(U) The functional data flow is best discussed in terms of four smaller subsections. Sections 2.2.1 and 2.2.2 are concerned with the data reduction necessary to produce the absolute measure of CAD model performance described in Section 2.2.3. Comparison of the CAD model performance with operator performance is discussed in Section 2.2.4.

2.2.1 Phase I Data Reduction

(U) Before the TECHEVAL data described in Section 2.1 could be used in the CAD validation process some preliminary data reduction was necessary. Figure 1 shows a condensed description of the data reduction process described below.

(U) The calibration data processing calculates the mean and standard deviation of the noise present using data from the first two echo cycles of the run. The calculated values of mean and standard deviation are passed to the thresholding process.

(U) The thresholding process uses a log likelihood ratio (see Appendix A) threshold to produce an array containing the value (in log likelihood ratio) and location of each peak that exceeds this threshold. This processing condenses the large volume of uniform time function samples by extracting peak sample events, and uses the mean and standard deviation obtained from the calibration data to transform these peak amplitudes to a normalized quantity, in this case log likelihood ratio.

(U) It is evident that this thresholding process is more involved than the normal thresholding process. The data is first scanned to select the local peaks. (A sample value x_i is a local peak if and only if $x_i > x_{i-1}$ and $x_i \geq x_{i+1}$). When two local

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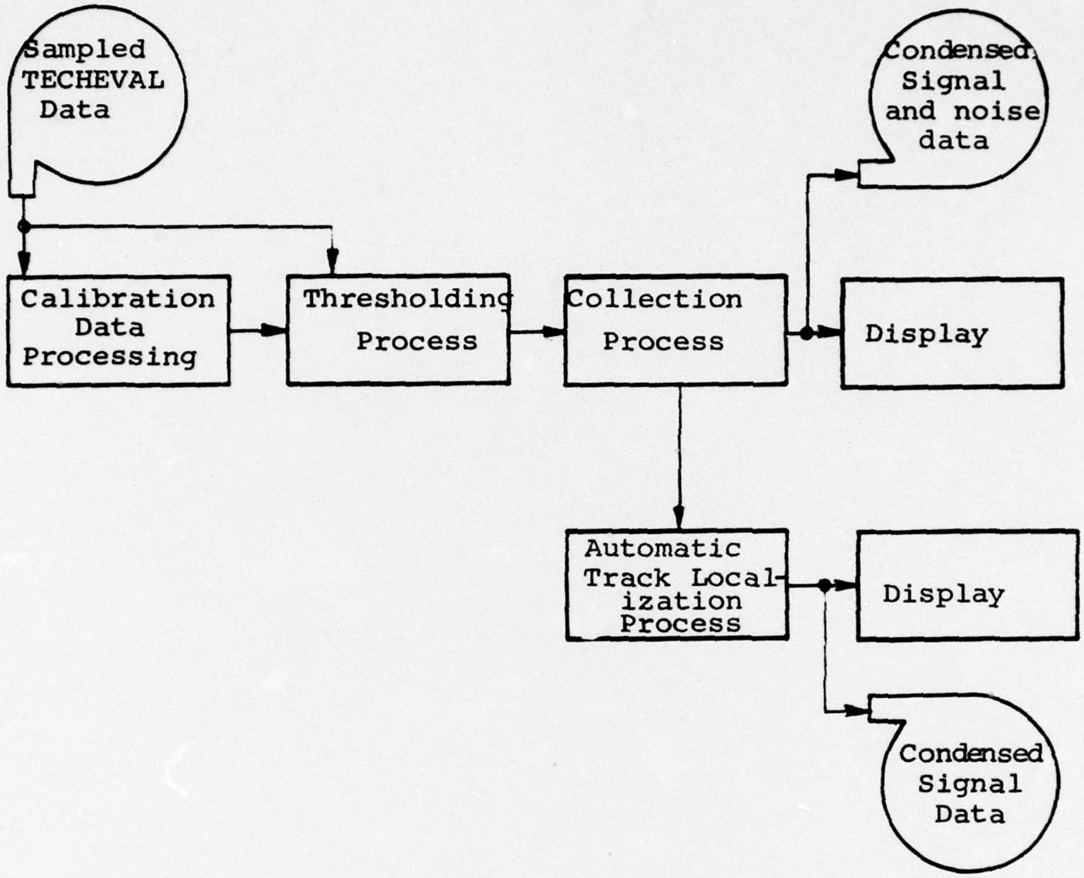

Fig. 1 Block Diagram of Phase I Data Reduction

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peaks occur within a single resolution interval of the sonar, the ambiguity is resolved by selecting the larger of the two. Each local peak is then converted to a log likelihood y_i through the relation

$$y_i = a\left(\frac{x_i - \mu}{\sigma}\right) + b \quad ,$$

where

μ = the mean value of the data calculated from the calibration data.

σ = the standard deviation of the data calculated from the calibration data,

a and b = the conversion constants for the log likelihood ratio L, L = ax + b.

The numerical values used for a and b are 2.45 and -5.2 respectively. These values are derived in Appendix B.

(U) After the log likelihood ratio conversion the thresholding process uses the time code data to express the position of each local peak as an arrival time relative to the transmit time of the signal. Thus, the entries in the array produced by this process consist of normalized peak amplitudes expressed in units of log likelihood ratios and corresponding peak positions expressed in milliseconds relative to the transmit time.

(U) The collection process collects the peak amplitudes and positions, generated ping by ping in the thresholding process, into one large array containing information about the entire run. The resulting array is stored onto magnetic tape

for later processing, and is also processed to obtain a computer generated display. Figure 2 is an example of this type of display. The numbers down the left side of the display identify the ping cycles. The horizontal displacement represents time of arrival or range. The digits within the display indicate dB in excess of some given signal-to-noise ratio value shown in the display heading. The purpose of this display is to verify visually the track chosen by the automatic track localization process described below.

(U) To this point the information stored in the array produced by the collection process includes both signal peaks and noise peaks. In order to produce the desired measure of CAD performance, it is necessary to separate the signal information from the noise information. The automatic track localization process is implemented to obtain this separation.

(U) In the automatic track localization process, peaks belonging to a signal track are extracted from the input array which is provided by the collection process. Since the noise and signal peaks input to the process were obtained from N consecutive pings, a maximum of N signal peaks and positions may define a single track. When the track does not contain a signal peak on one or more pings, there will be fewer than N peaks. Many pings, however, will contain more than one signal peak that could be associated with a single track. The automatic track localization process uses a multiple-step procedure to select a single peak from each ping for a given track.

(U) The first step consists of locating the largest peak in the entire input array. An initial assumption is made that this largest peak (called the "pivot" peak) belongs to the track. All peak positions and amplitudes on each ping

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Fig. 2 Display of Condensed Signal and Noise Data

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DIGITS BELOW ARE DB IN EXCESS OF   10 DB

    12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
  1    0        1       1            1          2    0
  2                               0                  6
  3  0              0       0                                          0
  4                0 1          1                    6
  5    0                   0                         2      1
  6                                                  1          0
  7                                                  5                          0          0
  8         1                                                        0
  9                             1                      2     0        0
 10                             0           0      0 4       1
 11             0    2                         0      6                              1                 0
 12                                                 34                            0
 13                                                             2
 14                                                        0                      0
 15                                                  2
 16                                                  2                          0           1            1
 17              2                                    6     0
 18
 19                                                  2                                            02
 20    2                                          0  2             1                         0
 21             2                      1              3
 22                     3                                                                         0
 23                                                  3                0   0                   0
 24                                                  2   3                                   3
 25    1                    0                        1                       0
 26            0                                      3
 27  0  1                                            3      0
 28                                                  3       0                                    2
 29  2                                               5
 30               1                    1              2
 31
 32                  1                                1
 33                                    0              2
 34                                                  2                        0             0
 35                                  1                                    20               1
 36                    1                          0  4                                 1          1
 37                                                      1                   3
```

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(excluding the ping containing the pivot peak) are then examined and an amplitude-weighted slope density function is calculated. The slope from each peak position to the pivot peak position contributes to the density function by an amount related to the amplitude of the peak. After all peaks have been examined, the slope corresponding to the maximum of the density function is selected and is used to define a straight line through the pivot peak. This straight line is the first approximation to the track.

(U) The straight line track approximation is used to obtain an initial set of signal peaks, which may initially include more than N peaks. A signal peak on a given ping is a candidate for inclusion in the track if its amplitude, degraded by the deviation of its position from the straight line approximation, exceeds a set threshold. To reduce the number of peaks in the set to N (or less), the largest such peak on each ping is selected to form the initial set of signal peaks.

(U) The remainder of the automatic track localization process consists of an iterative procedure in which successive least squares quadratic fits are made to signal peak positions beginning with the initial set of signal peaks described above. After each quadratic fit has been determined, the input array of peaks is examined to see which peaks belong to the track defined by the quadratic curve. At this point one or more of the initially-selected peaks may be replaced by other peaks which are closer to the curve. The peaks selected then become the basis for the next curve-fitting step. The process terminates when two successive quadratic fits define tracks containing the same set of signal peaks.

(U) At the termination of the automatic track localization process, the signal peak amplitudes and positions defining the track are produced in an output array. The data is stored onto a magnetic tape to be processed later and is processed to obtain a computer generated display to illustrate the selected track. From Fig. 3 it is seen that the format of this display is exactly the same as that of the previously described display. Therefore, comparison of the two displays to verify the results of the automatic track localization process is possible.

2.2.2 Phase II Data Reduction

(U) The next step in the validation process is to put the data into a form such that an absolute measure of CAD performance can be obtained.

(U) To produce the absolute performance measure two types of curves are needed:

(U) (1) False alarm rate curves as a function of the likelihood ratio threshold, and

(U) (2) Required signal-to-noise ratio for 50% probability of detection as a function of the likelihood ratio threshold.

The false alarm rate curves can be generated by processing a large volume of sea data noise through the CAD model. Required signal-to-noise ratio curves are generated from the signal data that has been processed by the CAD model. From the above brief discussion of the two types of information needed it is evident that it will be necessary to remove the signal from the signal plus noise data before any further processing can be accomplished.

(U) To aid in the discussion of the Phase II data reduction a block diagram is provided in Fig. 4.

Fig. 3 Display of Condensed Signal Data

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DIGITS BELOW ARE DB IN EXCESS OF   10 DB

    12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
  1                                                  0
  2                                                  6
  3
  4                                                  6
  5                                                  2
  6                                                  1
  7                                                  5
  8
  9
 10                                                  4
 11                                                  6
 12                                                  4
 13
 14
 15                                                  2
 16                                                  2
 17                                                  6
 18
 19                                                  2
 20                                                  2
 21                                                  3
 22
 23                                                  3
 24                                                  2
 25                                                  1
 26                                                  3
 27                                                  3
 28                                                  3
 29                                                  5
 30                                                  2
 31
 32                                                  1
 33                                                  2
 34                                                  2
 35
 36                                                  4
 37
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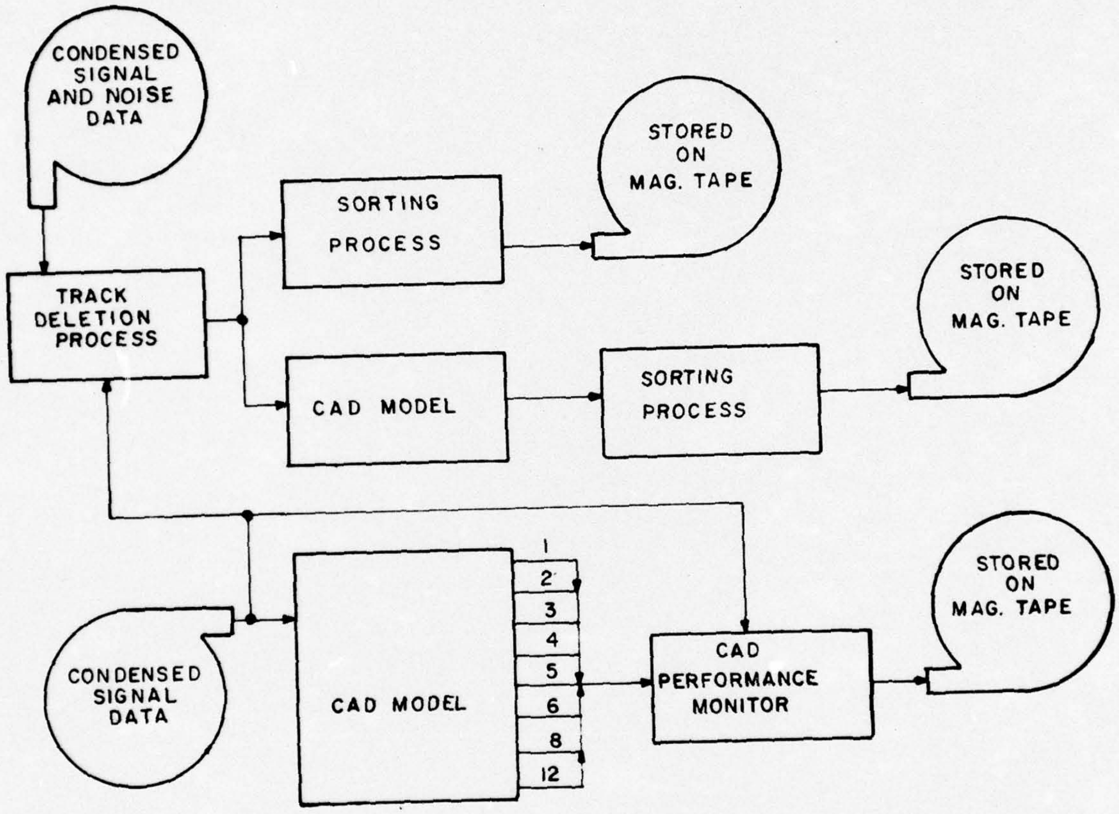

FIG. 4 BLOCK DIAGRAM OF PHASE II DATA REDUCTION

(U) The condensed signal and noise data consists of the amplitudes and location values produced by the collection process discussed earlier. From the automatic track localization process, also described in the previous section, the amplitudes and locations of the samples contributing to a track are known. The track deletion process uses this condensed signal data to remove the signal track from the signal plus noise array.

(U) Because of the sampling techniques used in digitizing the TECHEVAL data, the submarine track usually occurs about 1.5 seconds from the beginning of the cursor gate. Multiple paths to the target often exist, causing multiple parallel tracks. Because these multiple tracks are usually confined to the first five seconds of the cursor gate, these first five seconds are not used in developing false alarm rate curves. This procedure successfully eliminates most target tracks. As a further precaution, in processing the data for false alarm rate curves, any track which integrates to a log likelihood ratio of fourteen is called as a target track and is not counted in the noise false alarm statistics.

(U) The resulting noise only data is catalogued by the sorting process to obtain a tabular representation of the noise probability density function. This table is output to magnetic tape for later processing. The noise only data is also processed through the CAD model and into the sorting process to obtain noise probability density function applicable to the output of the CAD model. Likewise, this table is output to magnetic tape for later processing.

(C) As explained above, the probability of detection curves are produced from the results obtained by processing signals through the CAD model. Details concerning the CAD model may be found in Appendix A. For discussion here, it is

sufficient to state that the CAD model implements ping-to-ping integration of the logarithm of the likelihood ratio along consistent target tracks. The expected track strength (or track likelihood ratio) increases as the number of pings increases. This result is consistent with results obtained with optical ping-to-ping integration.

(C) From the discussion above, it is apparent that the probability of detection associated with a ping-to-ping integrating system depends on the number of pings for which integration is accomplished as well as on the average signal-to-noise ratio. It is also evident that the performance of the CAD model varies with respect to the number of pings the model is allowed to access. For this reason, to adequately evaluate the CAD model it was necessary to vary the number of pings to be integrated, and thus determine the model's performance when allowed to access 3 pings, 4 pings, and so forth.

(U) As mentioned earlier, the TECHEVAL data consisted of approximately 20 pings per run. The CAD model was modified to enable it to clear its memory after each sequence of N pings was processed in order to simulate having had access to only those N pings.

(U) One way to process these sequences would have been to determine the CAD performance individually for each value of N. However, in the interest of efficiency, all the sequences were processed through the CAD model each time a run was processed. The eight CAD model output channels shown in Fig. 4 contain information obtained by allowing the model to have access to different numbers of pings as shown on the output labels, 1, 2, 3, 4, 5, 6, 8, and 12.

(U) The CAD performance monitor uses CAD model input and output data to obtain information packages consisting of four entries:

(U) (1) N, the number of pings for which integration was performed.

(U) (2) LLR, the maximum track log likelihood ratio obtained for the N pings processed.

(U) (3) S/N, the estimated value of signal-to-noise ratio for the N-ping sequence going into the CAD model.

(U) (4) S/N;MAX, the maximum signal-to-noise ratio which occurred in the N-ping sequence going into the CAD model.

(U) The estimated signal-to-noise ratio refers to a signal-to-noise ratio of the form

$$\frac{P-\bar{x}}{\sigma} ,$$

where

P is the signal peak,
$\bar{x}$ is the noise mean,
σ is the standard deviation of the noise.

If the signal was detected on all N pings (i.e. no missed pings), then the estimated signal-to-noise ratio was a straightforward average. If some pings occurred which did not contain signals the average signal-to-noise ratio was calculated as,

$$\overline{\frac{S}{N}} = \frac{1}{N} \left[\sum_{i=1}^{K} \left(\frac{S}{N}\right)_i + (N-K) \left(\frac{S}{N}\right)_{est} \right] ,$$

where

K = the number of pings which contained a detected signal,

$(\frac{S}{N})_i$ = the signal-to-noise ratio for the pings which contained a detected signal,

$(\frac{S}{N})_{est}$ = an estimate of the signal-to-noise ratio for the missed pings. The value of $(\frac{S}{N})_{est}$ is calculated by a table lookup using $\frac{N-K}{N}$.

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3. DESCRIPTION OF RESULTS OBTAINED AFTER PROCESSING SEA DATA

(U) This chapter is concerned with a discussion of the results obtained using the processing procedures described in the previous chapter on sea data. A set of base line false alarm rate curves are described in Section 3.1, while Section 3.2 discusses the actual sea data false alarm rate curves. Section 3.3 provides a discussion of the absolute performance of the CAD model on target tracks. This performance is in turn compared with the performance of the operator on both single and multiple ping events in Section 3.4.

3.1 BASE LINE FALSE ALARM RATE CURVES

(U) To provide a basis for comparison of CAD effects on reverberation, a large set of noise was generated and passed through the CAD process. Initially 4.2×10^6 independent samples of Gaussian noise were obtained. This sequence was treated as a sampled time sequence with a sampling rate of 1,000 samples per second. The sampled time sequence was bandlimited by the application of a digital 5th order Butterworth filter with a 3 dB points at 100 and 200 Hz. The bandlimited noise was passed through a linear rectifier followed by a 10 sample finite-time perfect averager. The resulting "envelope" sequence was divided into 396 sections each containing 10752 samples. The data was first thresholded and sorted to obtain the curve of false alarm rate vs. log likelihood ratio shown in Fig. 5. Secondly, the data was processed through the CAD model and sorted to obtain the curve shown in Fig. 6. Each of these curves can be accurately represented by an expression of the form;

$$FAR = e^{A+B\cdot L+C\cdot L^2} \quad , \qquad (1)$$

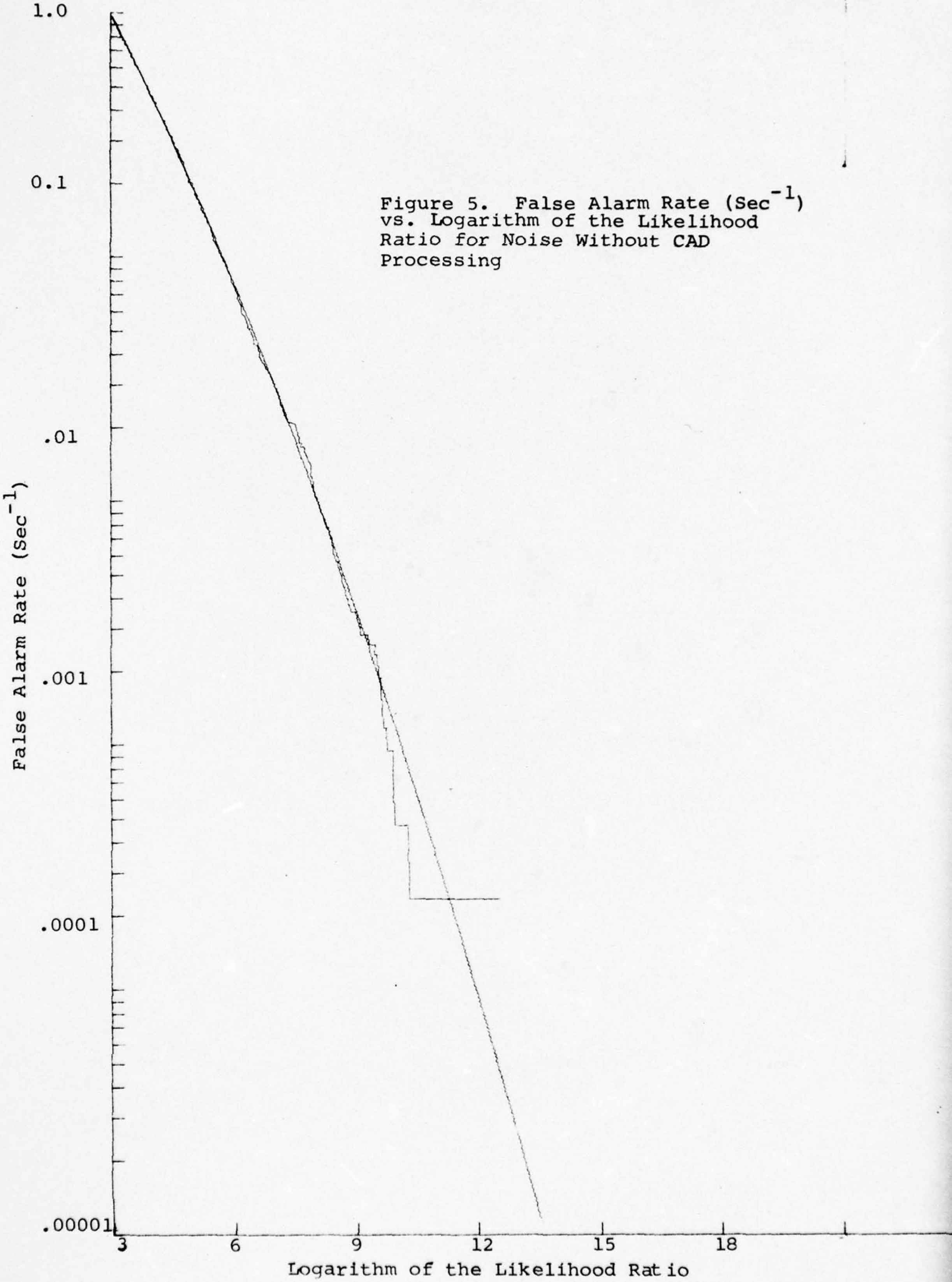

Figure 5. False Alarm Rate (Sec^{-1}) vs. Logarithm of the Likelihood Ratio for Noise Without CAD Processing

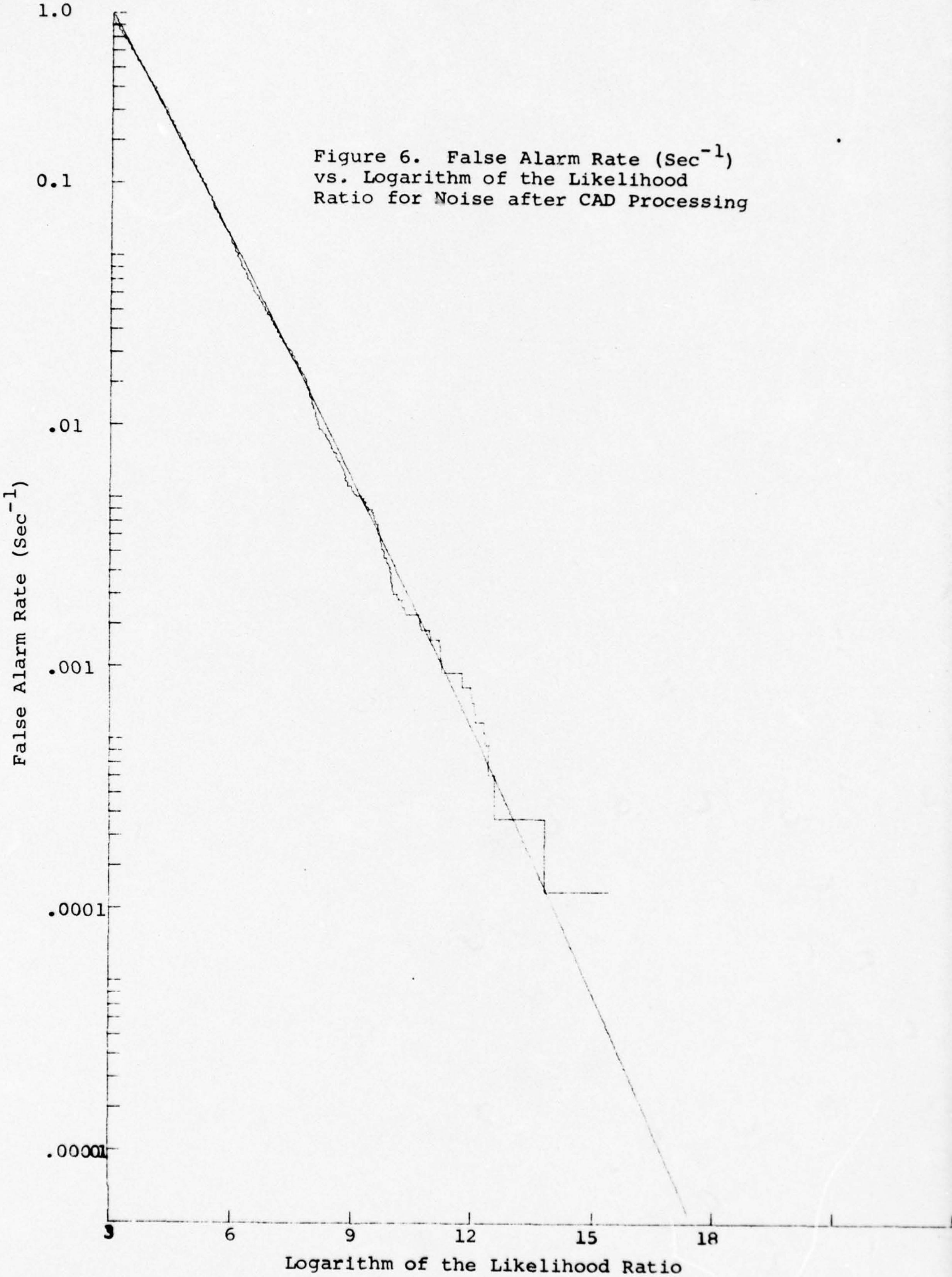

Figure 6. False Alarm Rate (Sec^{-1}) vs. Logarithm of the Likelihood Ratio for Noise after CAD Processing

where

FAR = False alarm rate, SEC^{-1},

L = log likelihood ratio, and

A, B, and C are constants.

The values of A, B, and C shown in Table I were determined by a least mean square fit process. The results obtained by evaluating Eq. (1) are plotted on Figs. 5 and 6.

TABLE I

FALSE ALARM RATE COEFFICIENTS FOR NOISE

		WITHOUT CAD	WITH CAD
	A	1.9289	1.8818
(C)	B	-0.5573	-0.5985
	C	-0.0315	-0.0130

(U) From the curves shown in Figs. 5 and 6 it is clear that the CAD model has adverse effects on the noise. This result is not unexpected and is more than compensated for by the benefits derived from tracking.

3.2 FALSE ALARM RATE CURVES OBTAINED USING SEA DATA

(C) Using data from approximately 1250 ping cycles of the TECHEVAL data base the false alarm rate curves with and without CAD, shown in Figs. 7 and 8, were generated. This data set represents approximately 10,000 seconds of sea data background. The false alarm rate coefficients are given in Table II. By comparing the curves of Figs. 7 and 8 to the base curves (derived from noise) in Figs. 5 and 6 it can be seen that the false alarm rates are slightly higher for sea data than for noise.

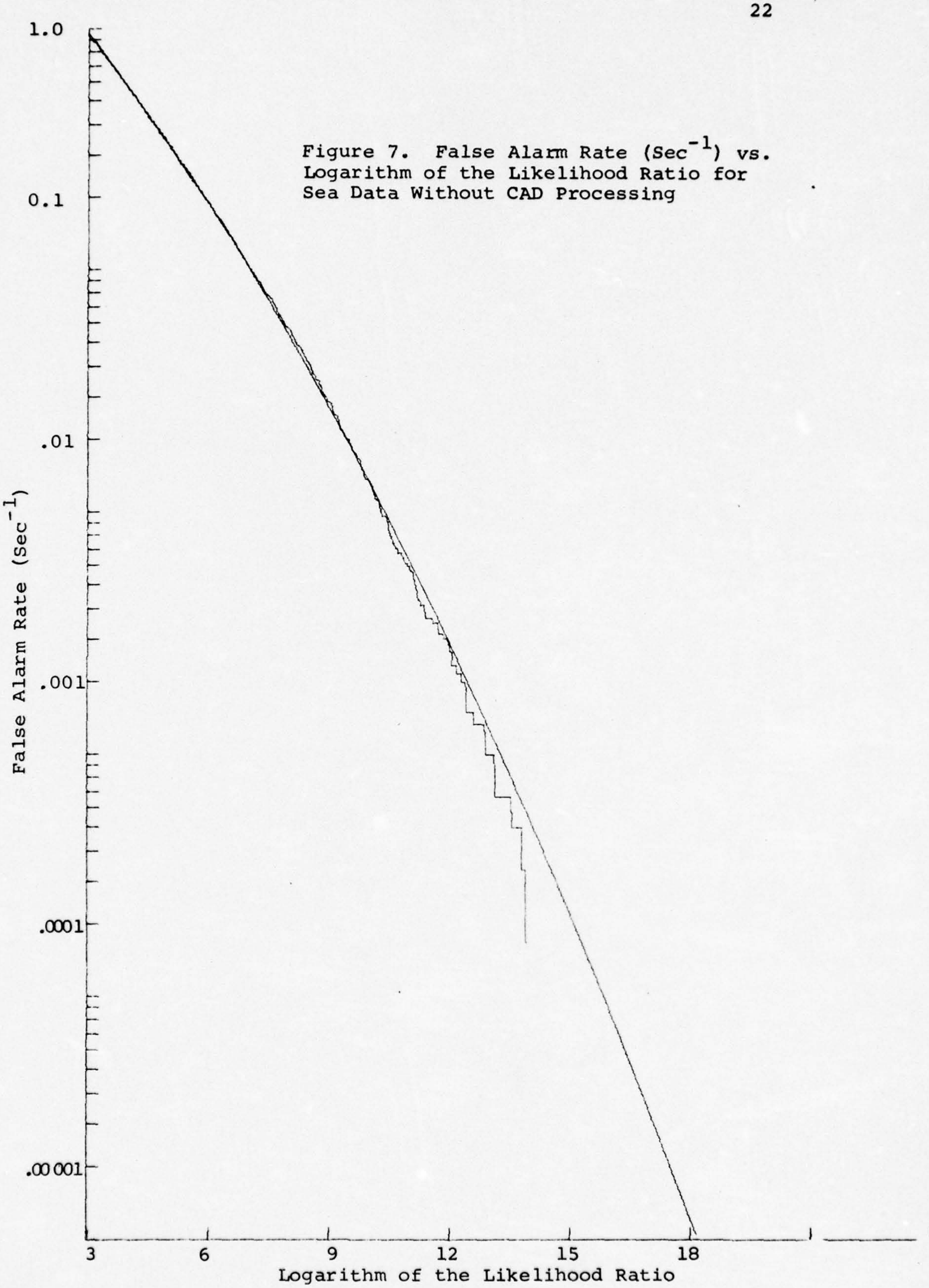

Figure 7. False Alarm Rate (Sec^{-1}) vs. Logarithm of the Likelihood Ratio for Sea Data Without CAD Processing

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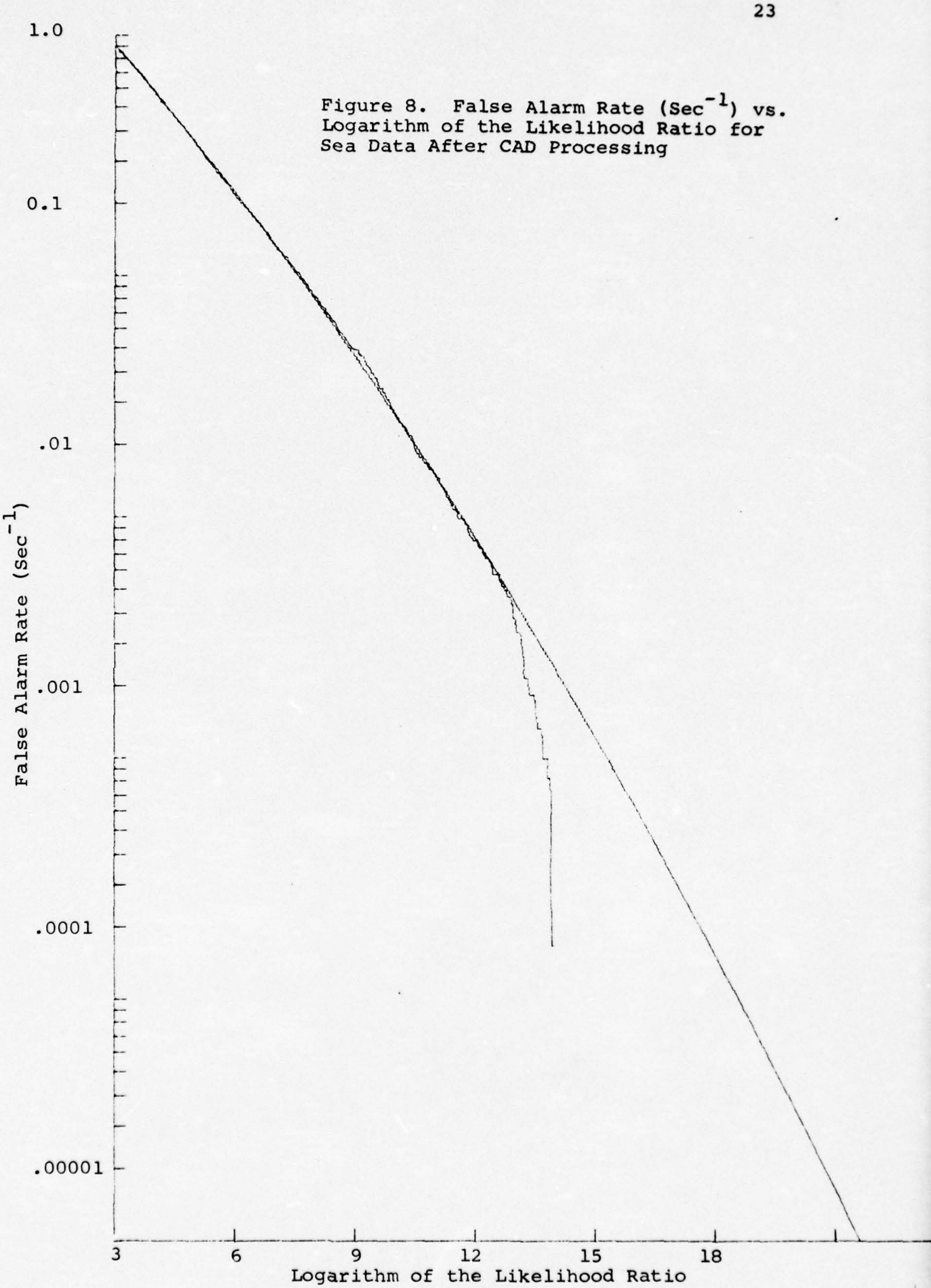

Figure 8. False Alarm Rate (Sec^{-1}) vs. Logarithm of the Likelihood Ratio for Sea Data After CAD Processing

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(U) In Fig. 8 a sharp cutoff of false alarms occurs at a log likelihood ratio value of 14. This cutoff is caused by the data processing procedure which automatically calls any track with a larger log likelihood ratio a signal track. To compensate for this artificial cutoff, the curve fitting procedure weights errors (i.e. deviations between the curve fit function and the measured false alarm rate function) with the false alarm rate. This weighting process causes the artificial cutoff of the measured function to have a very minimal effect on the fit curve since the false alarm rate is very low in the region affected by the cutoff. The resulting curve fit function should be more valid than that which would be obtained without imposing a cutoff on the measured false alarm rate curve.

TABLE II

FALSE ALARM RATE COEFFICIENTS FOR SEA DATA

		WITHOUT CAD	WITH CAD
	A	1.2254	1.0638
(C)	B	-0.3761	-0.3621
	C	-0.0178	-0.0107

3.3 ABSOLUTE PERFORMANCE MEASURE

(U) The performance of the CAD model on target tracks is condensed to a four entry track information package as described in Section 2.2.2. One track information package is generated for each track sequence processed. The four entries in the track information package are,

(U) (1) N, the number of pings which were processed,

(U) (2) LLR, the maximum track log likelihood ratio obtained by the CAD model,

(U) (3) S/N, the estimated value of signal-to-noise for the N-ping sequence going into the CAD model,

(U) (4) S/N;MAX, the maximum signal-to-noise ratio which occurred in the N-ping sequence going into the CAD model.

(C) By combining the information about CAD performance on target tracks with the false alarm rate information described in Section 3.2 it is possible to obtain an absolute measure of the total model performance. Several means are available for displaying this measure of the CAD model performance. One technique described in the first quarterly report* is to generate modified ROC curves. This technique requires a parameterization of the input signal-to-noise ratio. Since we have no direct control over the input signal-to-noise ratio, the required parameterization must be accomplished somewhat arbitrarily as described in the first quarterly report. To avoid this arbitrary parameterization, a different technique for displaying the results has been developed.

(C) For each track information package, the false alarm rate that would be associated with a threshold set so that the track sequence would be detected is determined by using the maximum track log likelihood ratio to index the false alarm rate curve associated with the output of the CAD model. Using this false alarm rate and the estimated value of signal-to-noise ratio, a point is plotted on a grid of signal-to-noise ratio

*Courts, H. R., "Processing Procedures for the Validation of a Computer-Aided Detection Model" (U), UNITECH 68-016-C, 30 July 1968, (CONFIDENTIAL).

vs. false alarm rate. This process is repeated for all available track information packages with a common value of N, the number of pings the model was allowed to use, to obtain a distribution of points on the grid. These points are fitted with a quadratic expression to obtain an estimate of input signal-to-noise ratio vs. false alarm rate.

(C) The results of this process are shown in Figs. 9 through 16, where the number of pings is varied parameterically. The signal-to-noise ratios are given in dB on the vertical scale and the logarithm (to the base 10) of the false alarm rate (in Sec^{-1}) is given on the horizontal scale. The symbol "X" is used to represent points derived directly from the track information packages and the symbol "." is used to plot the curve fit function. Many large signal-to-noise ratio tracks yield points which are off the scale of the graphs shown in Figs. 9 through 16. Even though these points are not shown, they are included in determining the coefficients of the curve fit since they do present valid information about trends.

(C) To provide some insight into the benefits derived from tracking it is interesting to determine what the performance would have been had the CAD model not been used. An alternate detection procedure would be to use the largest signal occurring in a track for detection, making no attempt to integrate along the track. If this were done, the appropriate false alarm rate curve would be the curve given in Fig. 7 without the CAD model. It is important to note that when several observations are allowed (such as 6) the expected value of the largest signal may be considerably larger than the average signal level. To obtain a display of the performance associated with this type of processing, the maximum signal-to-noise ratio which occurred

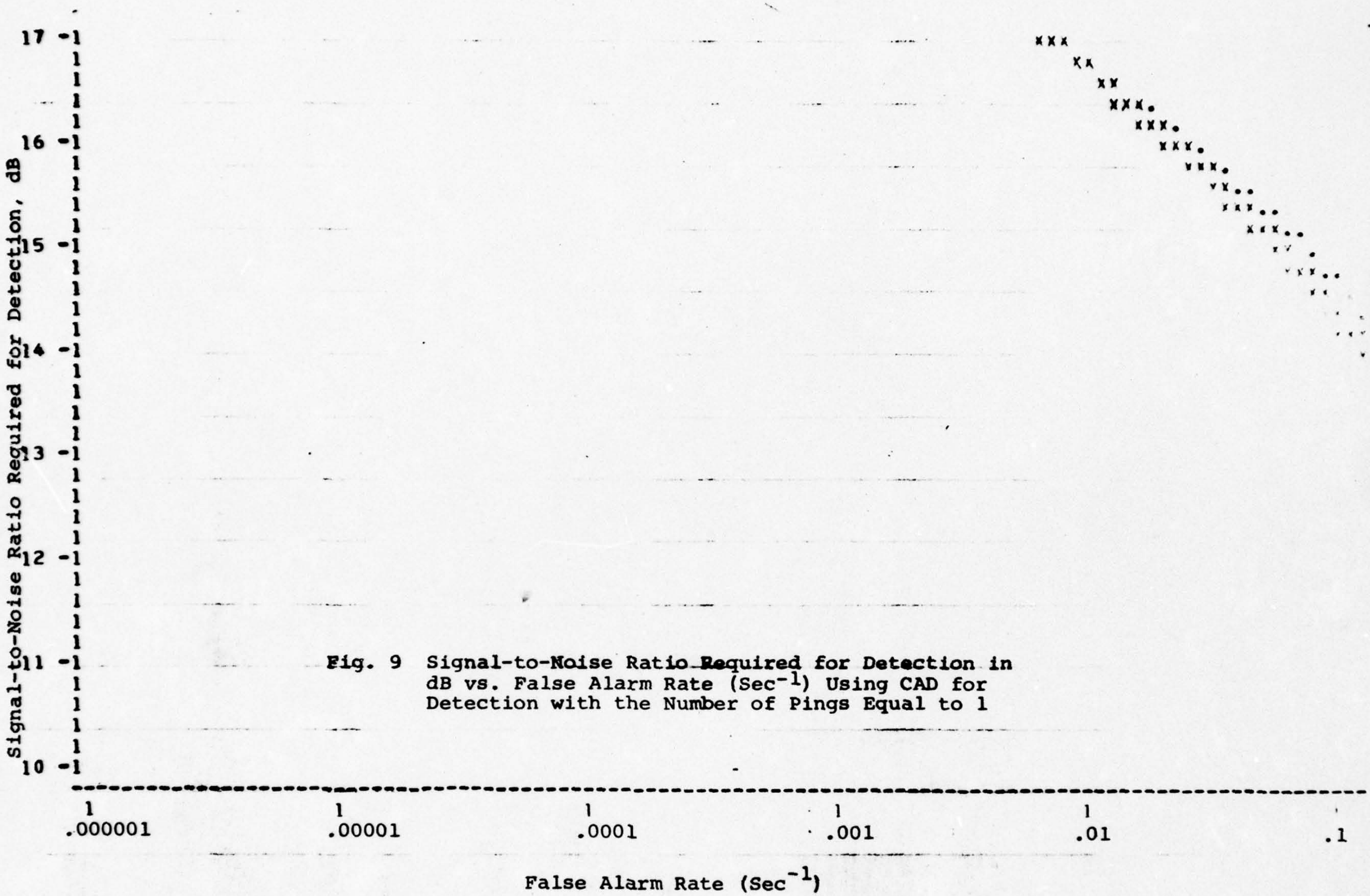

Fig. 9 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using CAD for Detection with the Number of Pings Equal to 1

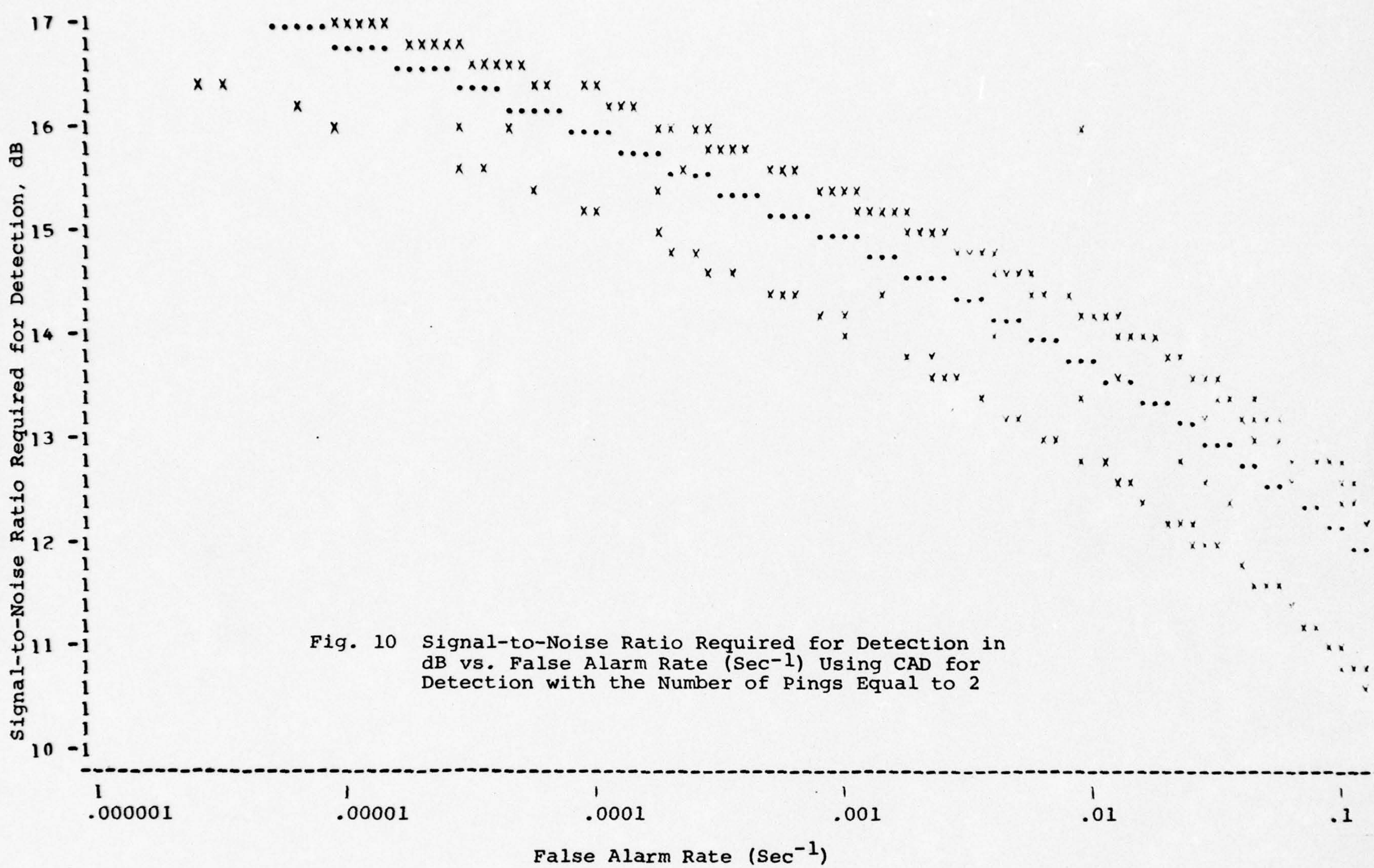

Fig. 10 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using CAD for Detection with the Number of Pings Equal to 2

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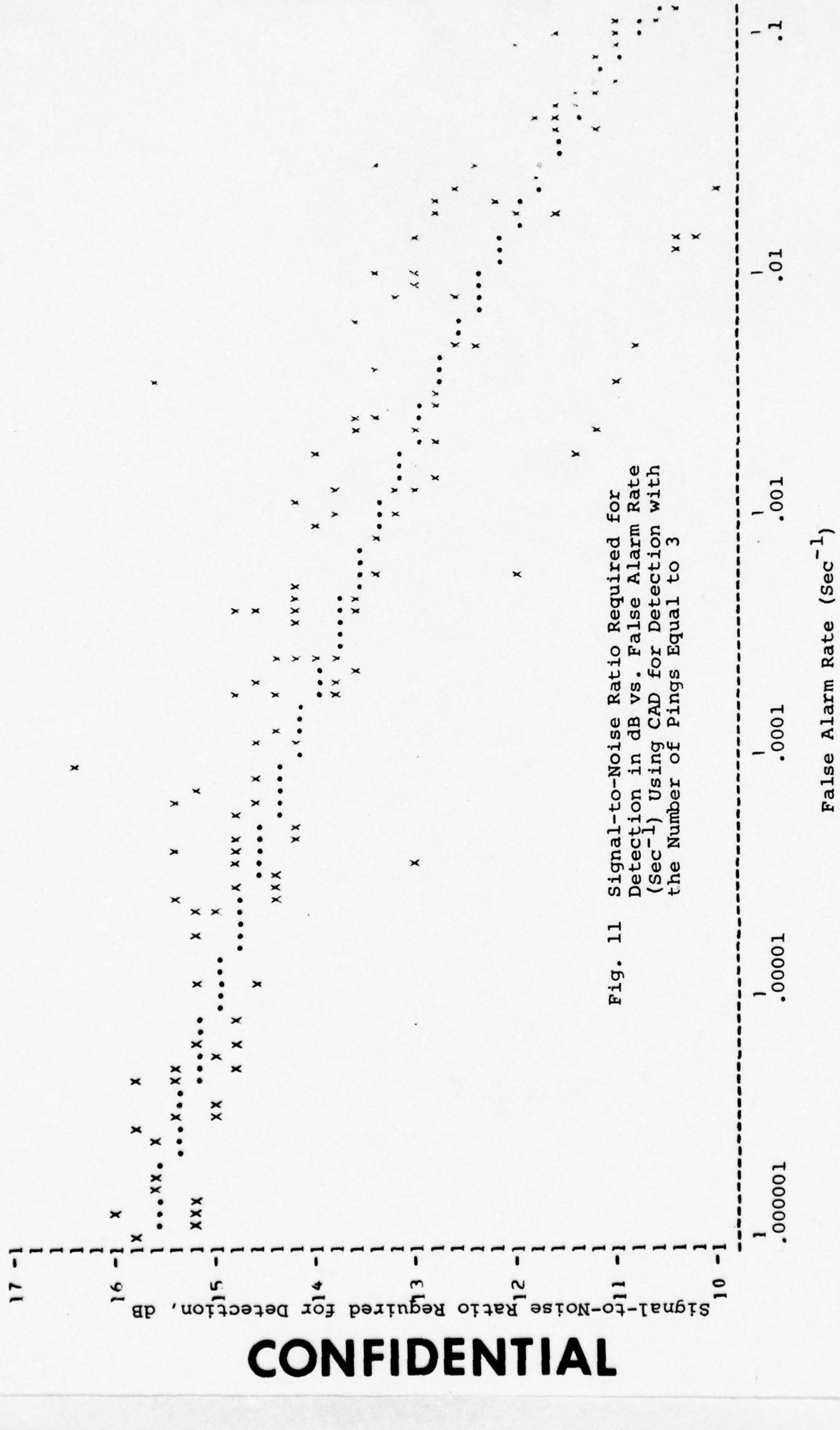

Fig. 11 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using CAD for Detection with the Number of Pings Equal to 3

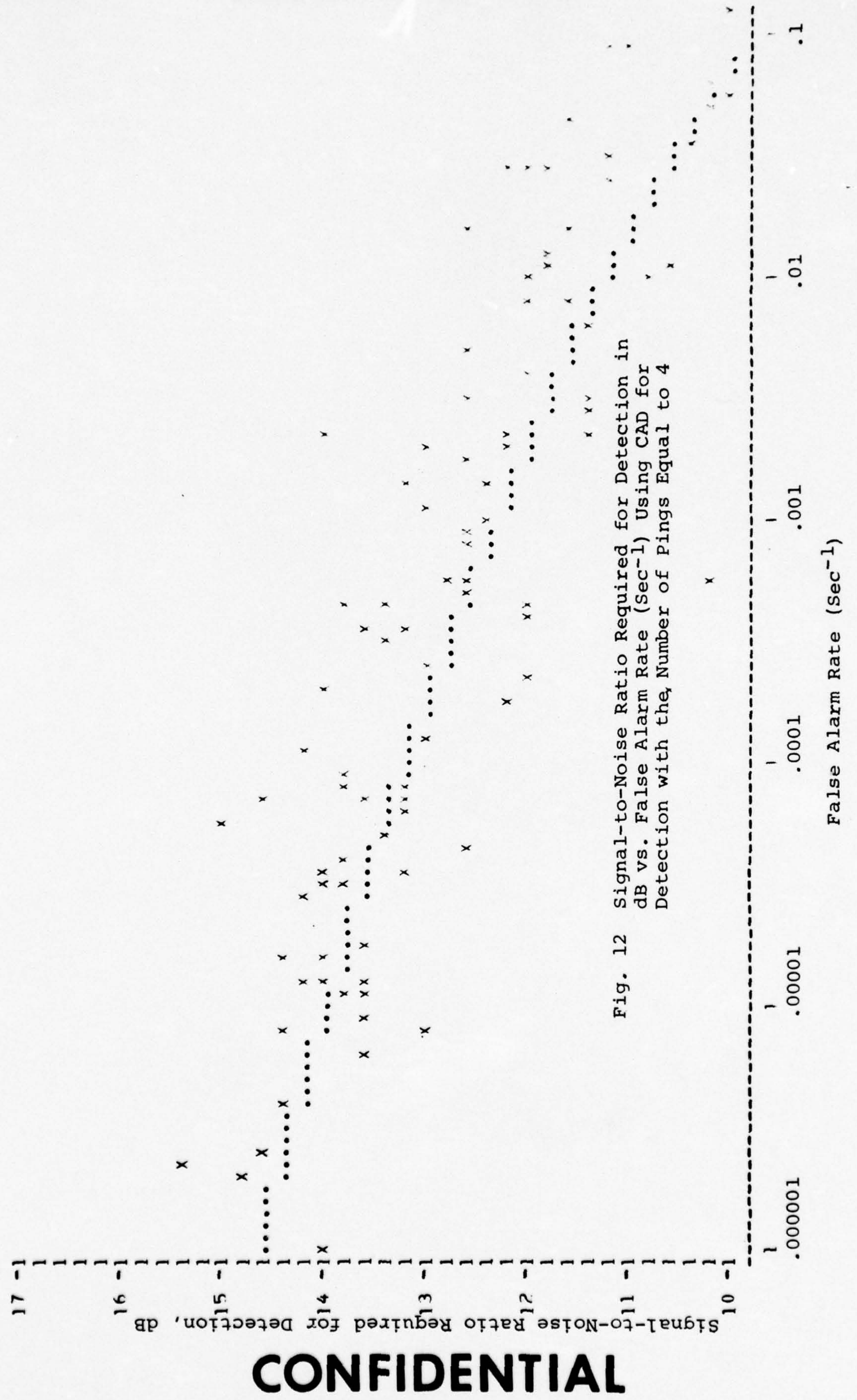

Fig. 12 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using CAD for Detection with the Number of Pings Equal to 4

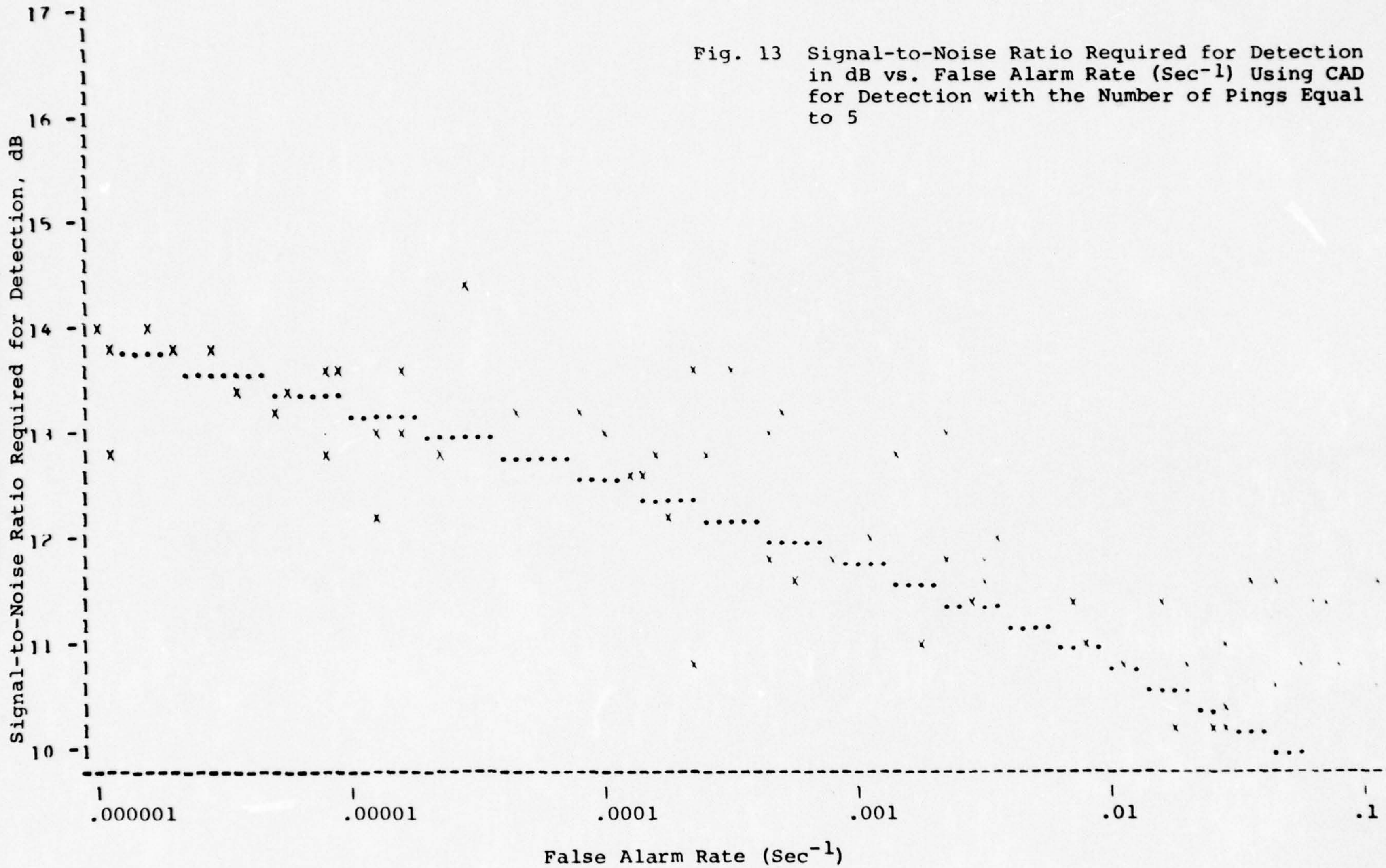

Fig. 13 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using CAD for Detection with the Number of Pings Equal to 5

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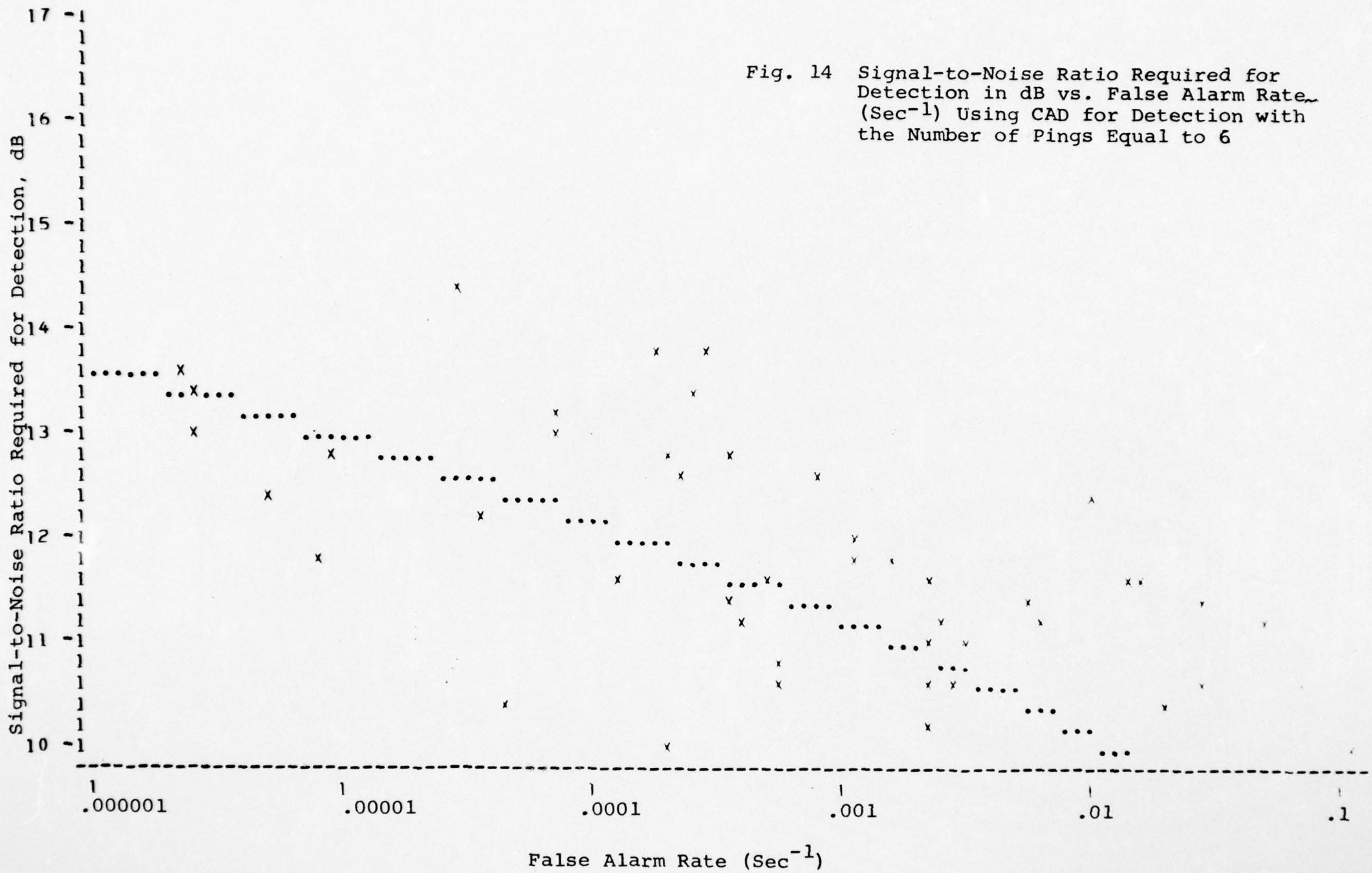

Fig. 14 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using CAD for Detection with the Number of Pings Equal to 6

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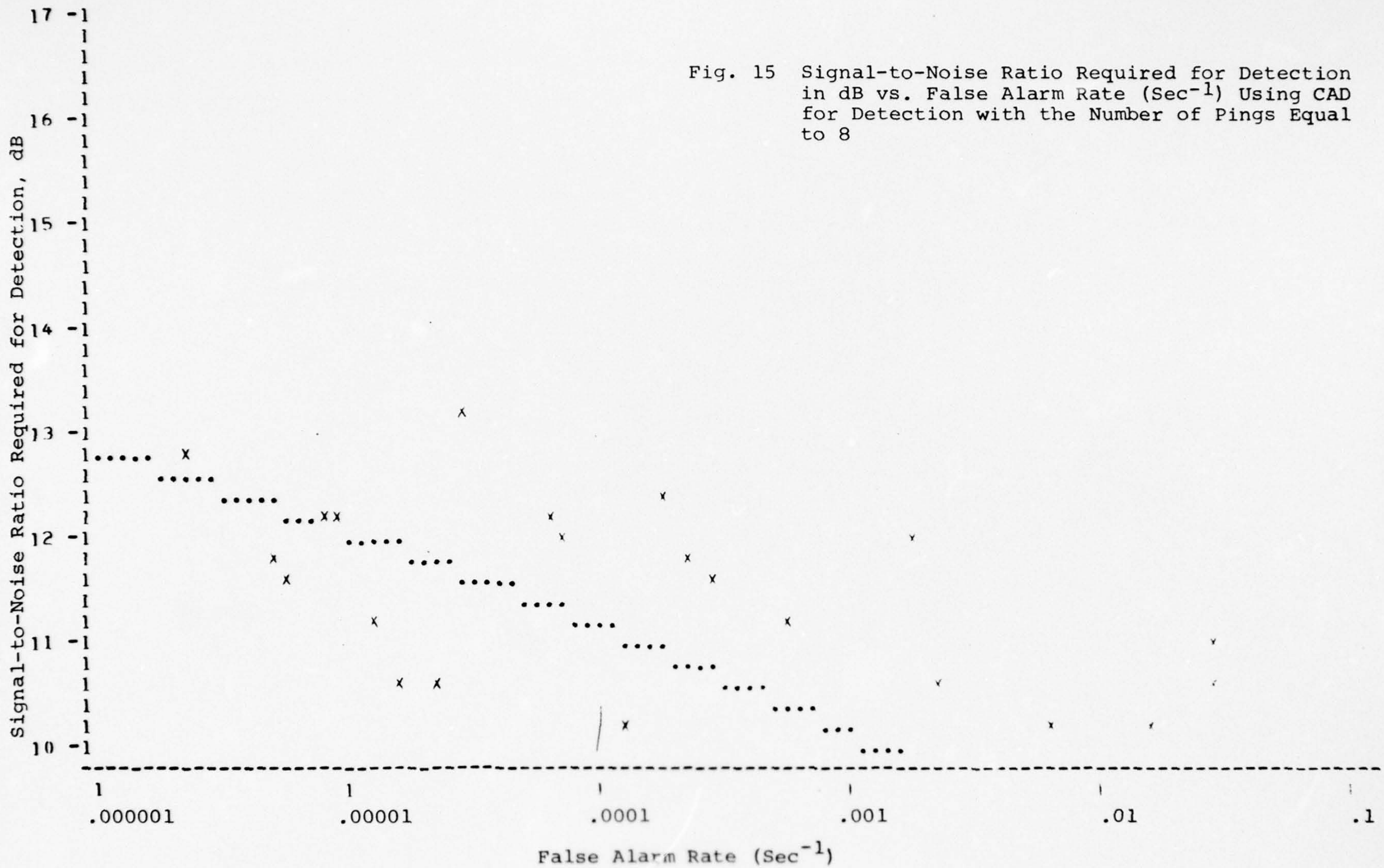
Fig. 15 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using CAD for Detection with the Number of Pings Equal to 8



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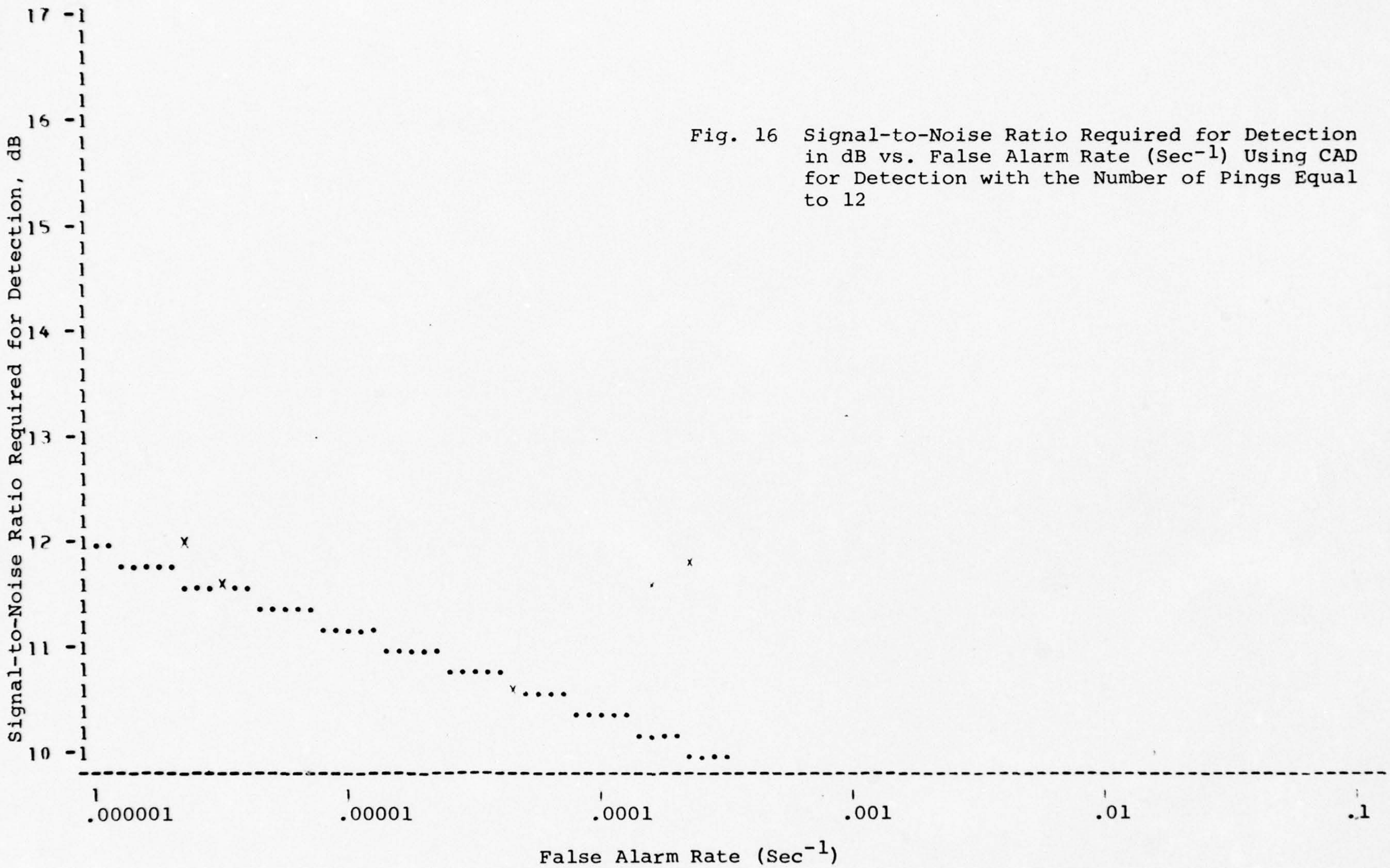

Fig. 16 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using CAD for Detection with the Number of Pings Equal to 12

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was transformed into a log likelihood ratio and used to index the false alarm rate curve without CAD to determine a false alarm rate. The resulting (FAR, S/N) pairs were processed as described above to obtain the results shown in Figs. 17 through 24. By comparing the results using CAD for detection with the corresponding results using the peak for detection, two conclusions can be drawn:

(C) (1) In an average sense the detection performance obtained with the CAD model is about 1 to 2 dB better than detection performance obtained using the peak signal for detection in the low false alarm rate region.

(C) (2) The detection performance obtained with the CAD model is much more consistent than that obtained using the peak signal for detection.

3.4 COMPARISON OF CAD PERFORMANCE WITH OPERATOR PERFORMANCE

(U) In addition to the absolute measure of CAD performance described in Section 3.3, it is desirable to obtain a performance comparison relative to the sonar operator. Section 3.4.1 provides a single ping performance comparison while Section 3.4.2 provides multiple ping performance comparison.

3.4.1 Single Ping Comparison of CAD Performance with Operator Performance

(C) As described in Section 2.1 the operator responses were logged for each ping cycle in the TECHEVAL data. The possible responses were: N-No Signal, W-Weak, M-Medium, and S-Strong. These four response levels provide information about three operator threshold levels, i.e. Strong, Medium or Better and Weak or Better. By combining the operator response data with the measured signal log likelihood ratios, three curves of

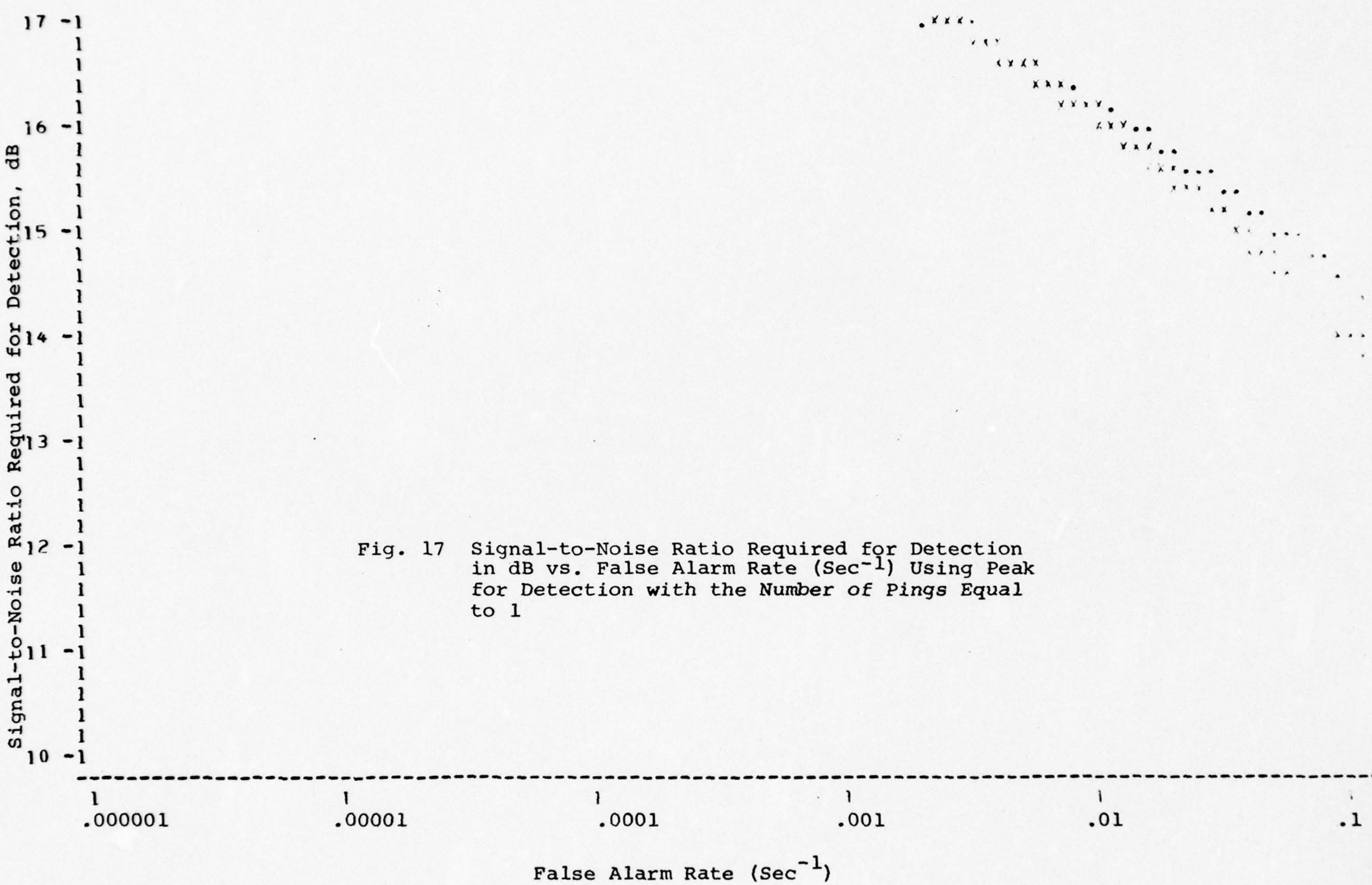

Fig. 17 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using Peak for Detection with the Number of Pings Equal to 1

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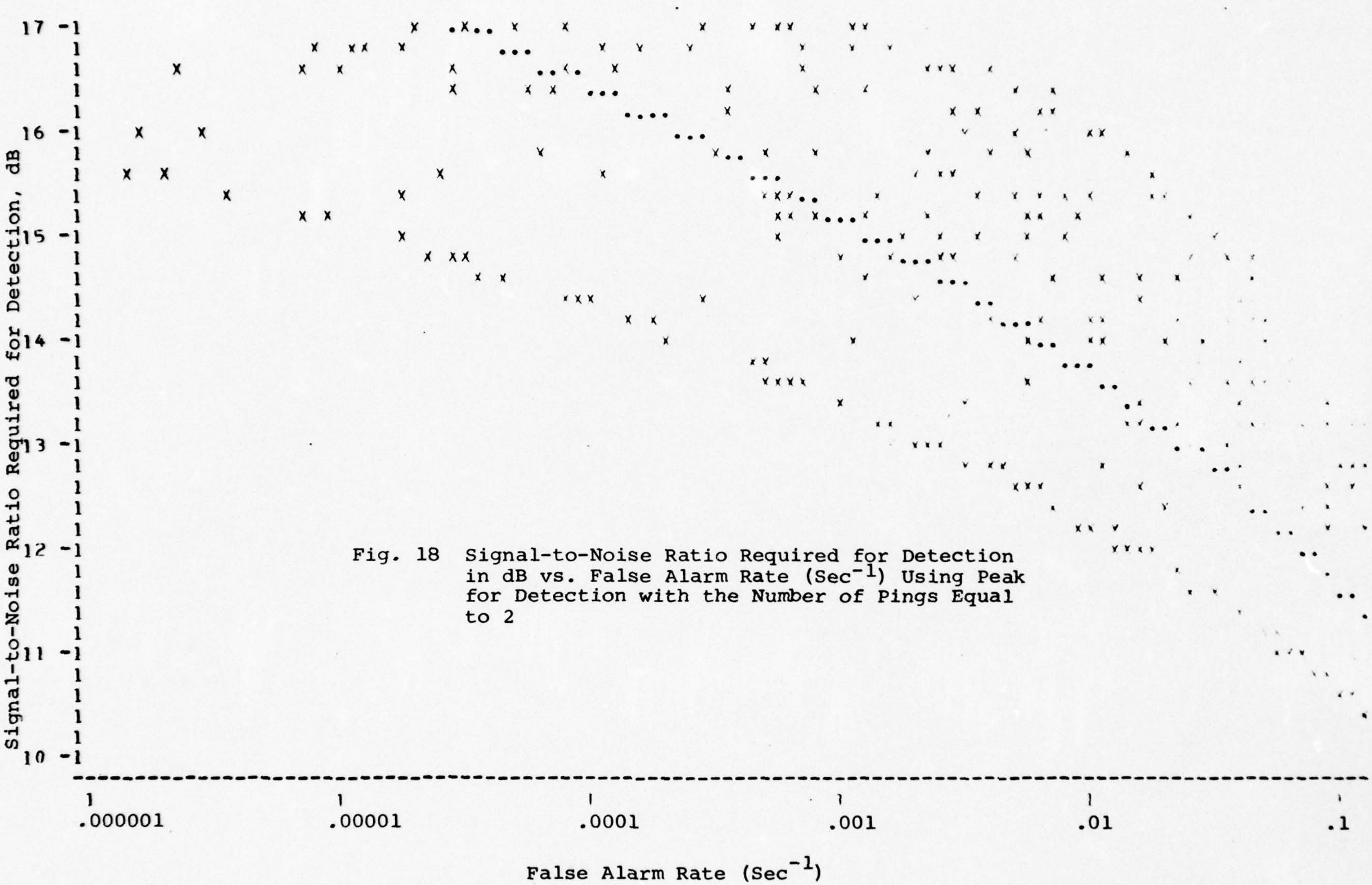

Fig. 18 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using Peak for Detection with the Number of Pings Equal to 2

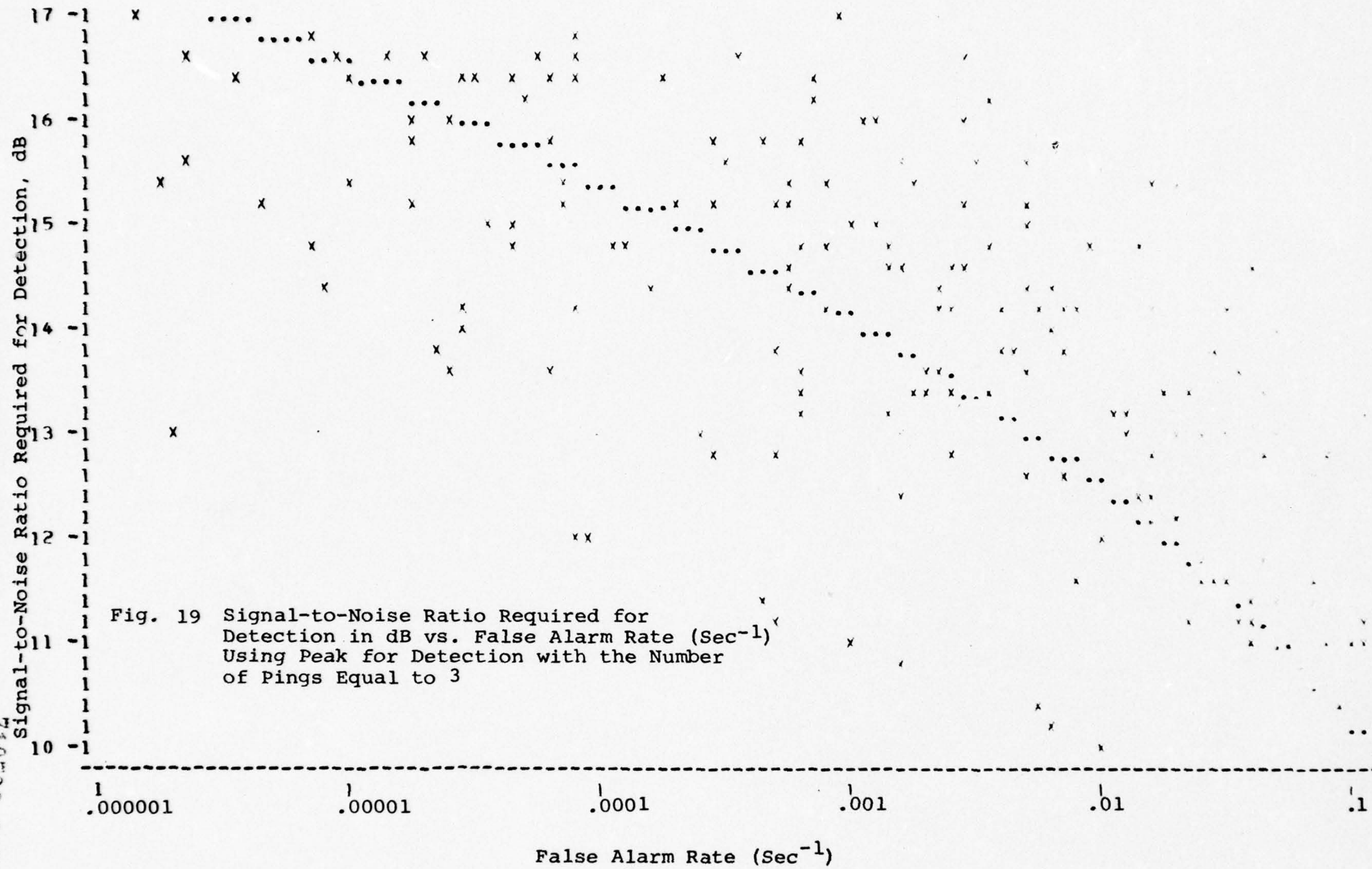

Fig. 19 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using Peak for Detection with the Number of Pings Equal to 3

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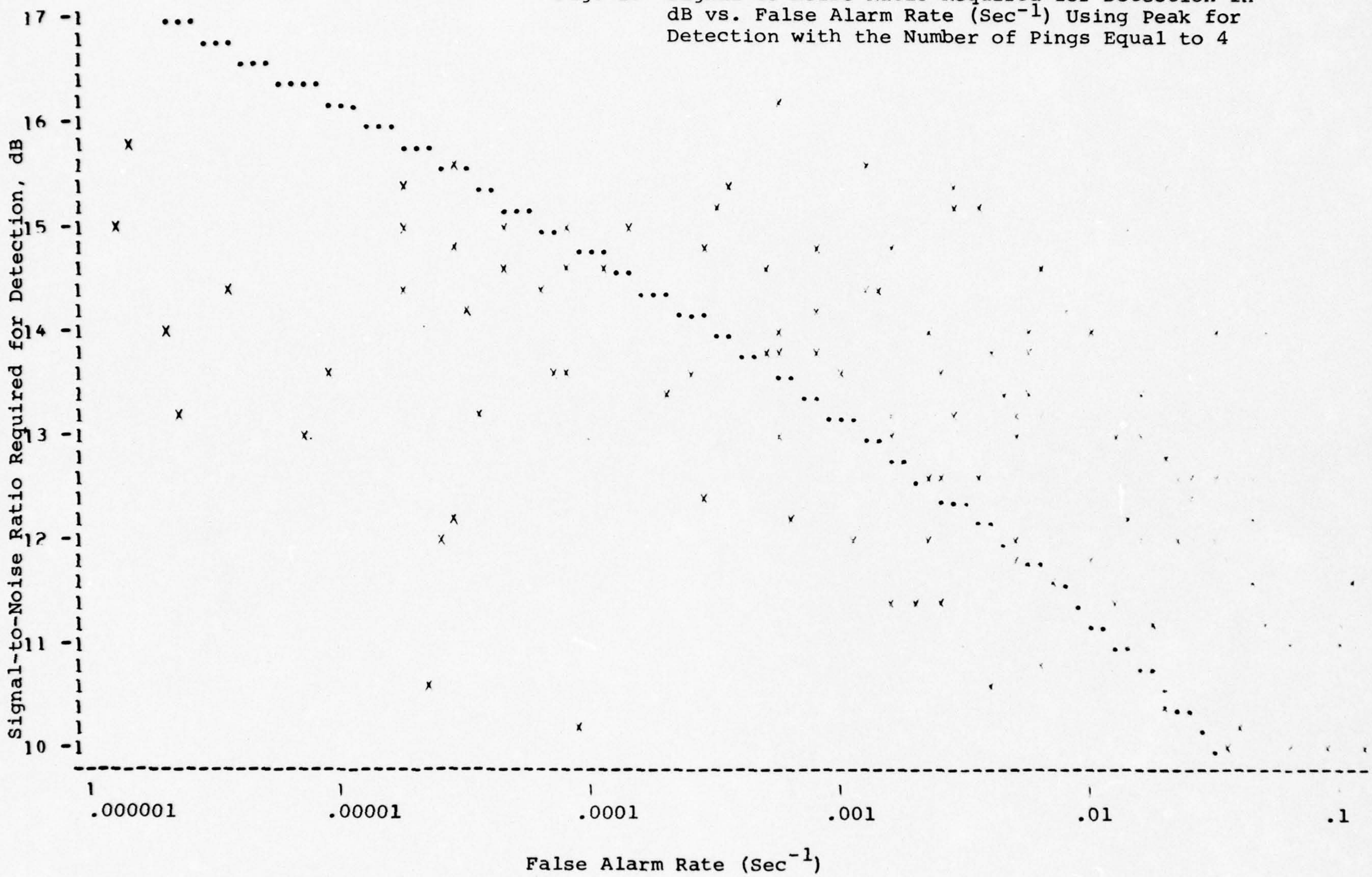

Fig. 20 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using Peak for Detection with the Number of Pings Equal to 4

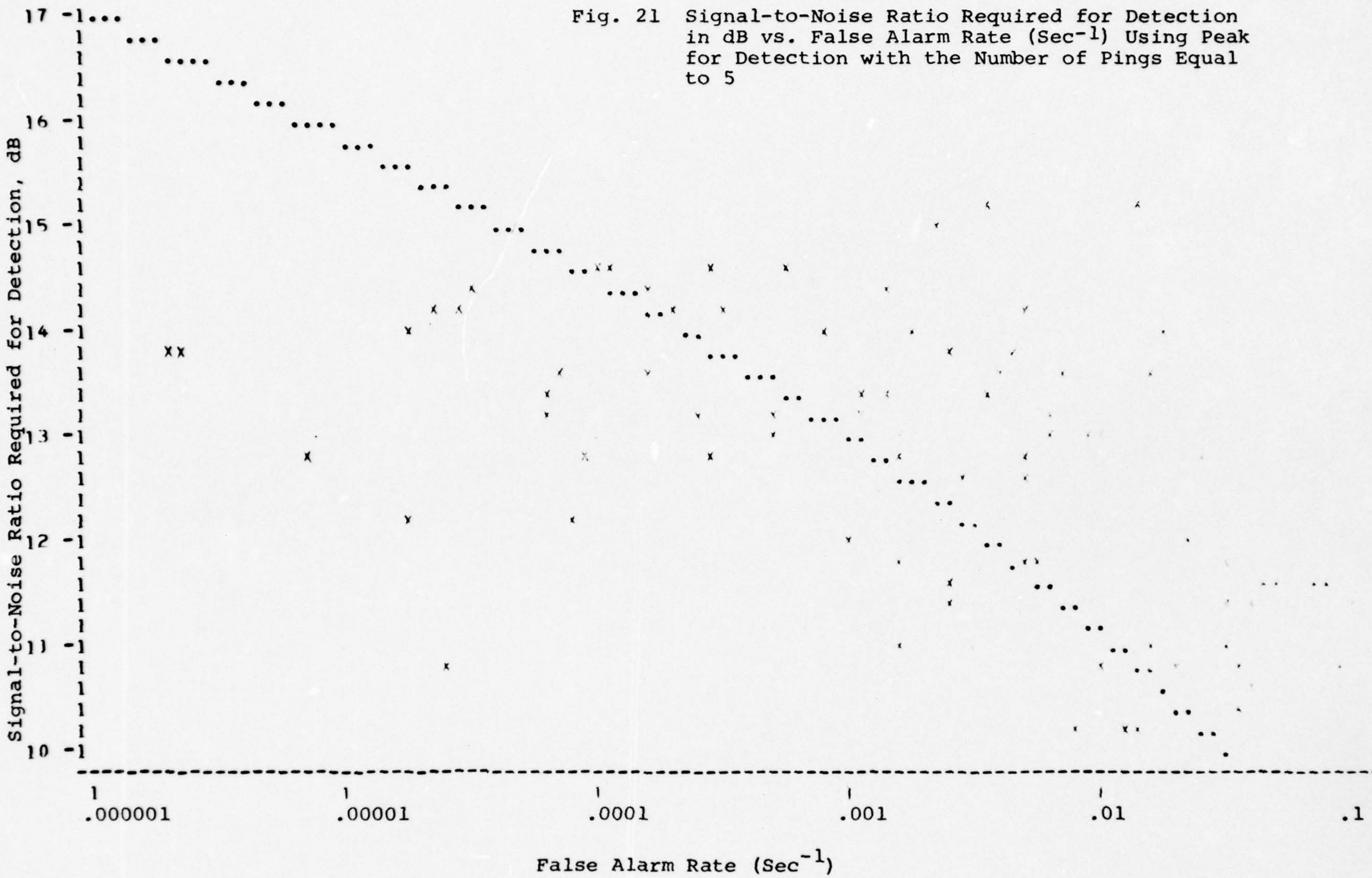

Fig. 21 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using Peak for Detection with the Number of Pings Equal to 5

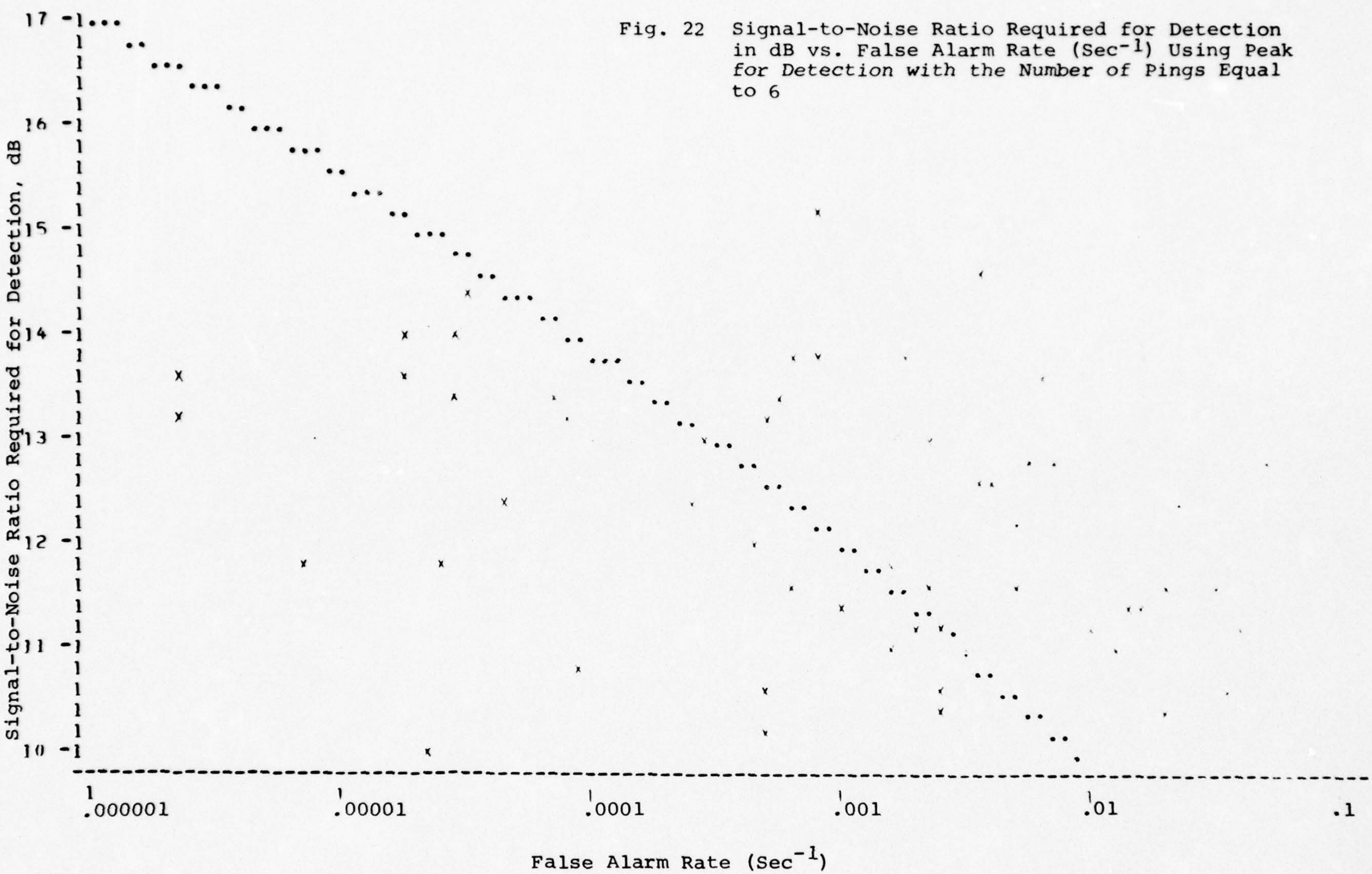

Fig. 22 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using Peak *for Detection with the Number of Pings Equal* to 6

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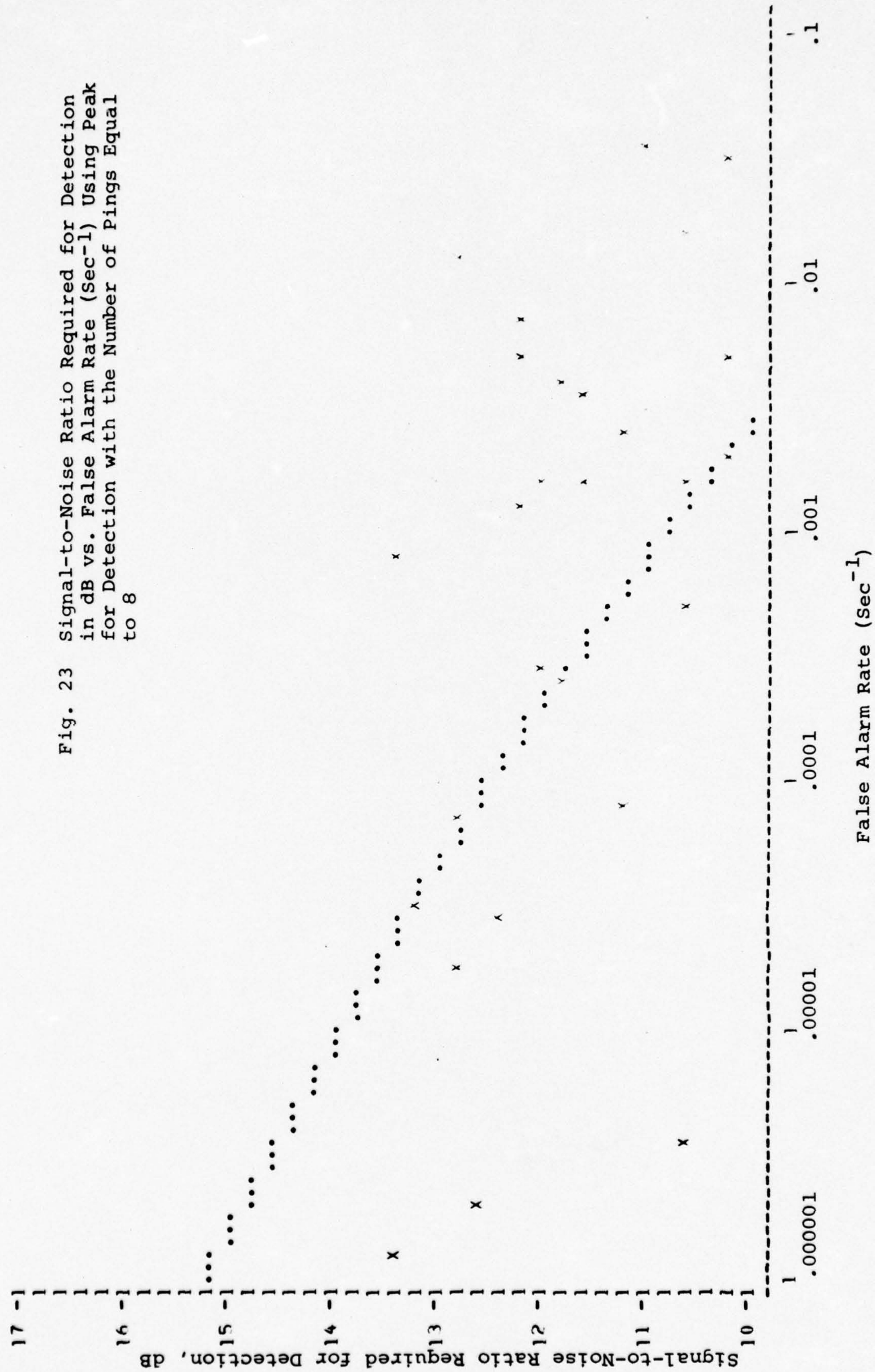
Fig. 23 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using Peak for Detection with the Number of Pings Equal to 8



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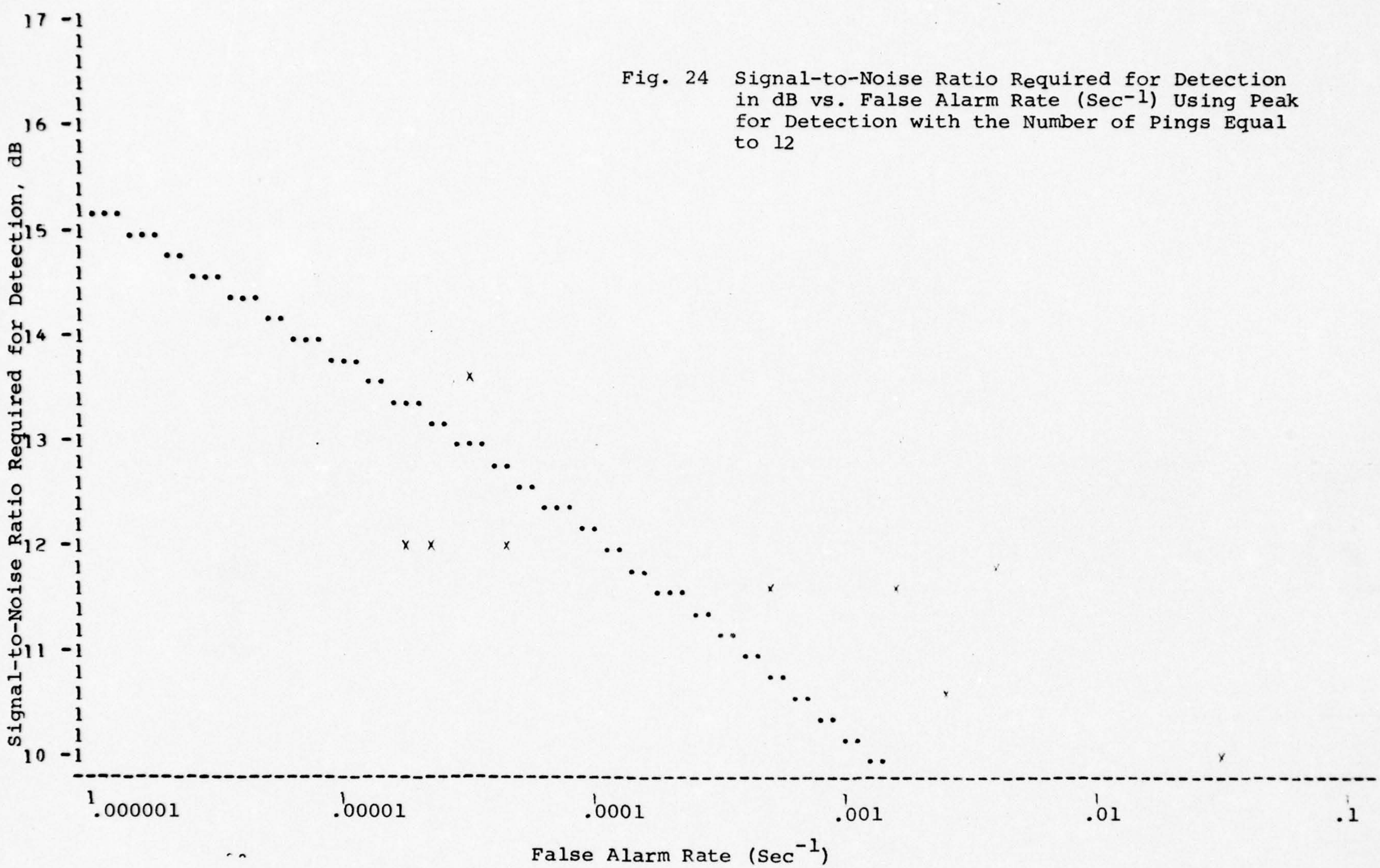

Fig. 24 Signal-to-Noise Ratio Required for Detection in dB vs. False Alarm Rate (Sec^{-1}) Using Peak for Detection with the Number of Pings Equal to 12

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probability of detection vs. log likelihood ratio are obtained. The three curves obtained from processing approximately 1200 operator responses are given in Fig. 25.

(C) The extreme variability of the operator response is especially evident in the distribution of strong signal calls. Some signals with a signal-to-noise ratio as low as 13 dB were called and other signals with a signal-to-noise ratio as large as 22 dB were not called strong. In processing the operator response data it appeared that the responses were very consistent in a relative sense within a run but not consistent in an absolute sense from one run to the next. This tendency toward relative rating of signals within a run may have occurred intentionally in the data gathering, in which case the summary results presented in Fig. 25 do not represent a good measure of the operator's ability to perform an absolute rating.

(U) It is possible to associate a false alarm rate with each of the operator response threshold levels through the relation,

$$(FAR)_{\emptyset P} = \int_0^{\infty} P_D(\ell) \cdot \left[\frac{dFAR(\ell)}{d\ell}\right] \cdot d\ell \quad ,$$

where

$FAR(\ell)$ is the single ping false alarm rate function described earlier, and

$P_D(\ell)$ is one of the three curves describing operator performance.

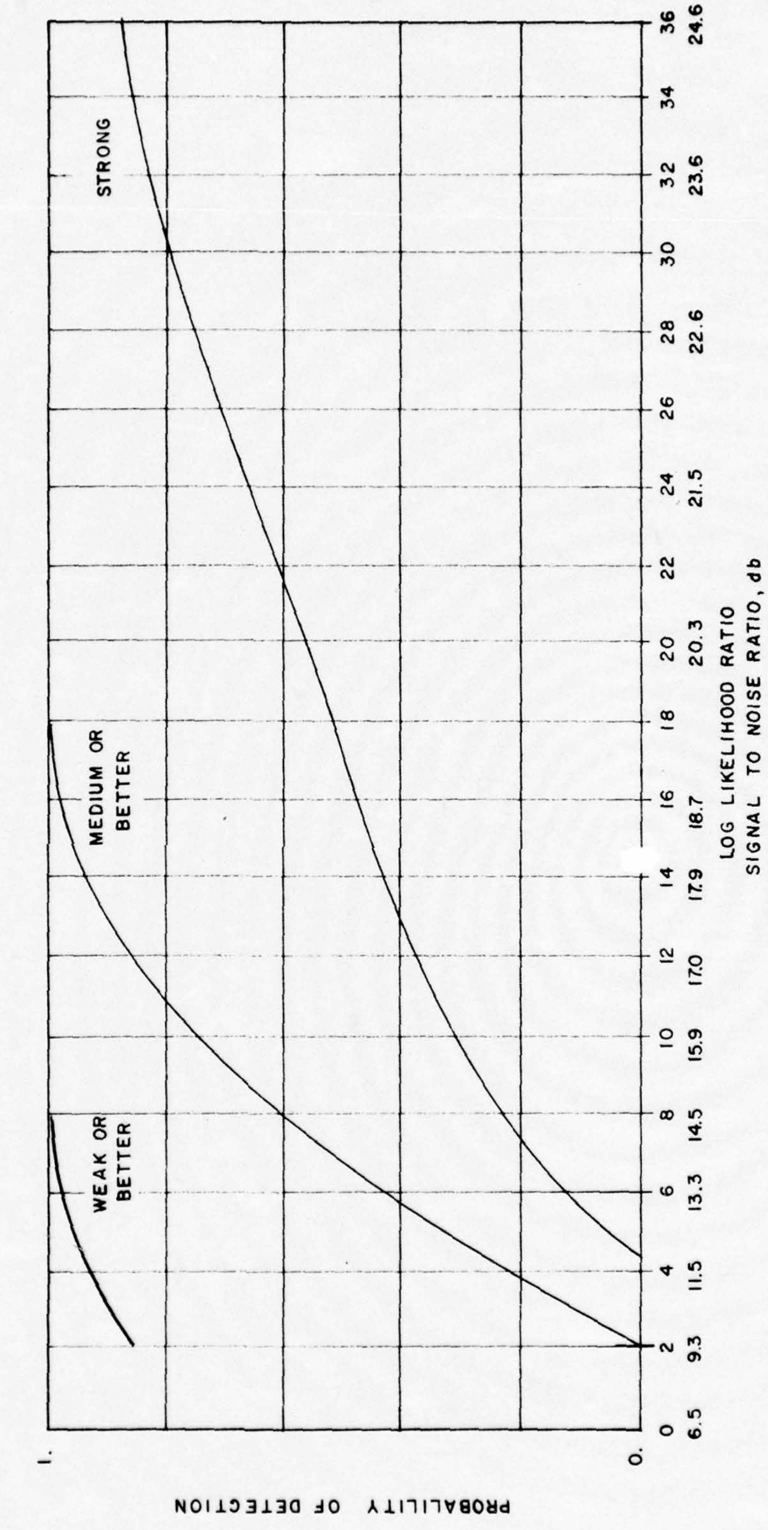

Fig.25 OPERATOR SINGLE PING PERFORMANCE RESULTS

In words, $\frac{dFAR(\ell)}{d\ell} \cdot d\ell$ is the differential rate at which noise events occur with log likelihood ratios between ℓ and $\ell + d\ell$, and P_D is the probability that the noise event will be called when it does occur.

(U) The false alarm rates associated with the three operator response thresholds may be compared with the false alarm rates associated with a perfect threshold device set to obtain 50% probability of detection at the same log likelihood value achieved by the operator. This approach provides the desired single ping comparison of the CAD model with the operator in terms of a difference in false alarm rate at the same probability of detection. These results are summarized in Table III.

TABLE III

SUMMARY OF OPERATOR RESPONSES

Response Threshold	Operator False Alarm Rate	50% Detection Level	Threshold Detector False Alarm Rate	$\frac{(FAR)_{ØP}}{(FAR)_{TH}}$
Weak	1.30	LLR=2, S/N=9.3 dB	1.5	0.8666
Medium	0.290	LLR=8, S/N=14.5 dB	0.06	4.8333
Strong	0.0276	LLR=13, S/N=17.4 dB	0.0015	18.40

3.4.2 Multiple Ping Comparison of CAD Performance with Operator Performance

(C) From the CAD performance curves described in Section 3.3 a curve of decrease in signal-to-noise ratio required for 50% probability of detection (from that required when N=1) vs.

number of pings (N) of integration was generated. This curve is shown in Fig. 26. Curves of 10 log N, 7 log N, and 5 log N are included for reference. The curve labeled "OPERATOR" was obtained from Fig. 3-53 of the AN/SQS-26CX Sonar Signal Processing Review.*

*Sonar Signal Physics Committee, "AN/SQS-26CX Sonar Signal Processing Review," (U), 1 March 1966, (CONFIDENTIAL).

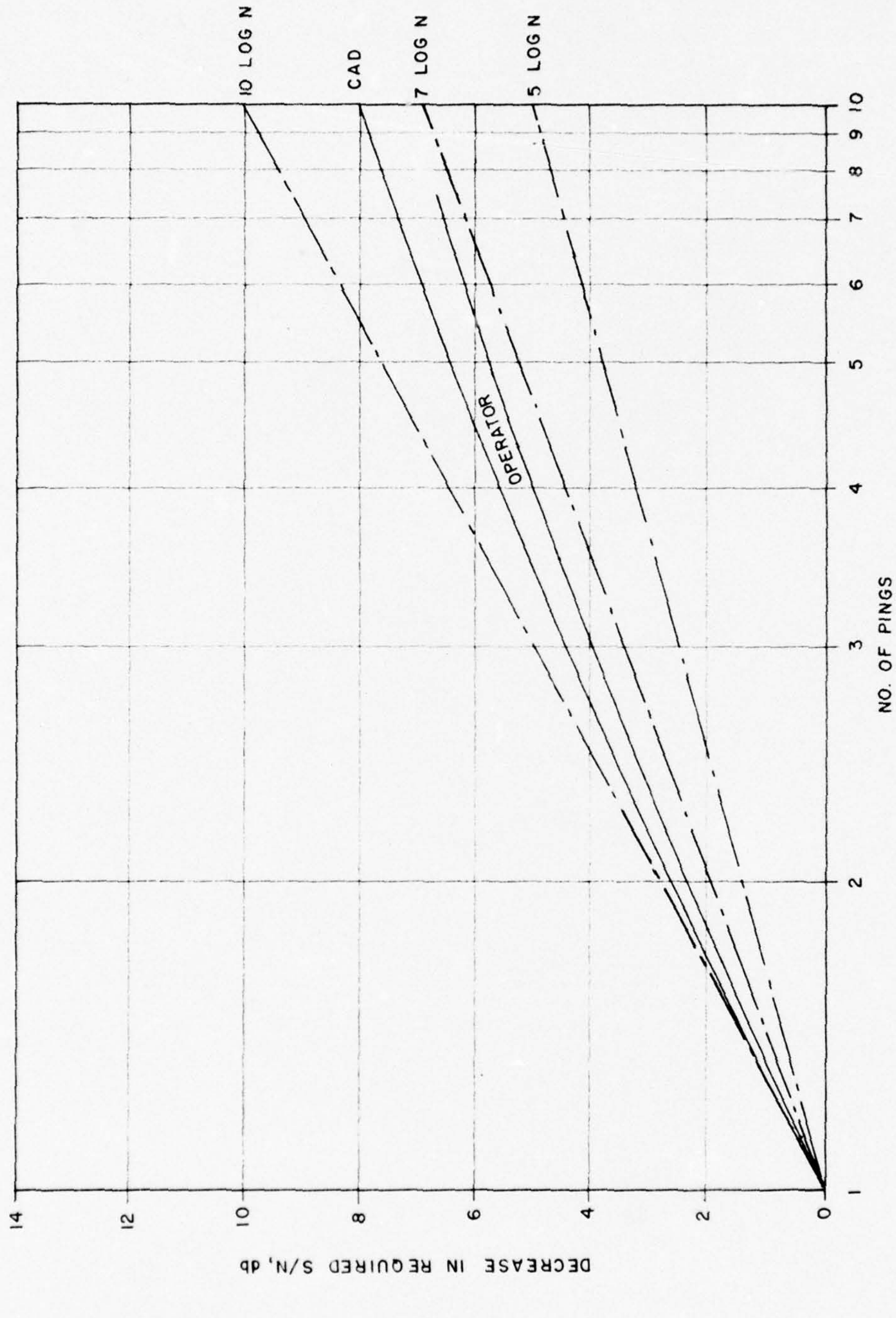

Fig. 26 MULTIPLE PING COMPARISON OF CAD PERFORMANCE WITH OPERATOR PERFORMANCE

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APPENDIX A. DESCRIPTION OF THE CAD MODEL

A.1 THE CAD MODEL

This appendix is intended to provide some insight into the computer-aided detection model.

The logical flow of data within the CAD model is shown in Fig. A1. The natural time base for an active track detection model is the ping interval, depicted in Fig. A1 by the layers labeled echo cycles N-1, N, and N+1. The CAD model uses two different types of input information, that stored in a master status file and that coming from the sonar signal processor.

Information pertaining to possible target tracks is stored in the master status file in the form of multiple ping event packages. Each of these packages consists of data which was obtained from one or more ping cycles and which shows promise of defining a target track. During echo cycle N, single ping event packages are formed from ping N and used as input to the model. When viewed in this way it is seen that the goal of the model is to combine single ping event packages from each echo cycle in an optimal way with multiple ping event packages from the master status file to produce an updated master status file. The information in the updated master status file is then passed to the next echo cycle for use as input by the track detection model during that echo cycle.

The track strength of each possible track represented in the status file can, during any echo cycle, be tested against a threshold, and if the threshold is exceeded the information in the multiple ping event package can be used to drive a display.

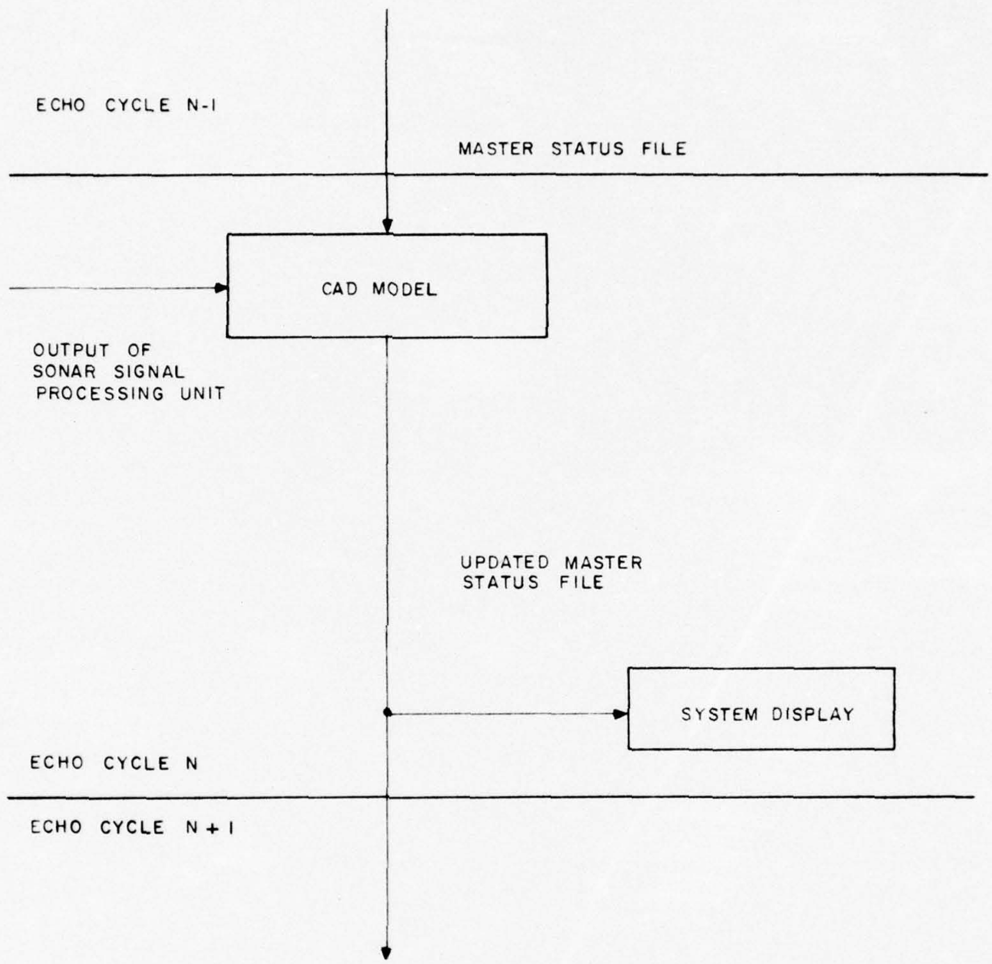

Fig. A-1 LOGICAL FLOW OF DATA

A.2 EVENT PACKAGES

A.2.1 Single Ping Event Packages

The main function of the CAD model is to combine the single-ping information received from a sonar signal processing unit with the information from previous pings to produce an updated master status file. The first step in this process is the conversion of processor output into single-ping event packages.

Information received during an echo ranging cycle is divided into single ping event packages by the application of a threshold to the output of the sonar signal processing unit. It is often possible to pick a threshold sufficiently low to pass any useful signal and still sufficiently high that 95-99% of the noise will not exceed the threshold. The single ping event packages contain the range position of the event and the single ping log likelihood ratio associated with the event.

The likelihood ratio, as applied to single events, is defined as the ratio of the probability that the event peak is a result of signal divided by the probability that it is a result of noise. The specific transformation from peak height to log likelihood ratio is dependent on the type of signal processing used and the average signal-to-noise ratio expected when a signal is present. Often the exact equation for transformation from peak height to log likelihood ratio is quite complicated. The transformation equations for a replica correlator and for the statistical wave period processor have been investigated in some detail, however, and it has been found that in the area of interest a linear transformation from peak height to log likelihood ratio is very accurate.

As mentioned above, the specific transformation also depends on the signal-to-noise ratio expected when a signal is present. To specify this quantity the minimum signal-to-noise ratio which will permit detection in a reasonably small number of echo cycles is used. In this way the system is optimum for signals near the minimum detectable level. The derivation of the transformation from peak height to log likelihood ratio is presented in Appendix B.

A.2.2 Multiple Ping Event Packages

As mentioned above, information pertaining to possible target tracks is maintained in the master status file. It is important to note that the storage required does not depend on the duration of a target track but only on the number of possible tracks under investigation at a given time. The number of tracks under investigation is controlled by a retention threshold employed in the main processing loop of the CAD model. A possible target track is defined in computer memory as long as its track likelihood ratio exceeds the retention threshold. (The likelihood ratio is the ratio of the probability that the events which compose the track represent signal divided by the probability that they represent noise.) In the validation study a likelihood ratio of 3 was used as the retention threshold.

A possible target track is defined by a multiple ping event package in the master status file. Each package contains the following four functional quantities used to describe a possible target track:

(1) Logarithm of the track likelihood ratio, TLLR.

(2) Range position of the latest processor output peak to contribute to the track, LR.

(3) Expected range position in the next ping of a peak belonging to the track, ER.

(4) Previous ping bit sequence.

The track likelihood ratio provides a measure of the track strength. To avoid the multiplication involved in track likelihood ratio computations, it is convenient to work with the logarithm of the likelihood ratio. When a new signal is received and linked with a track, the track likelihood ratio is updated by multiplying the old track likelihood (that of the multiple ping event) by the single signal likelihood ratio (that of the single ping event), then dividing by a loss factor dependent upon the deviation of the single signal's range position from the range position expected for a track sample.

The expected range position of the track on a given ping is determined by linear extrapolation from the position of the track peaks on the previous two pings. The expected range position and the previous range position of a track can be combined to produce an estimate of target range rate.

The previous ping bit sequence is a string of 24 bits. Each bit represents a previous ping and is set only if that ping contains a signal peak which belonged to the target track.

A.3 MASTER STATUS FILE UPDATING

The next stage in the combination of single-ping information with multiple-ping events is to update the master status file and thus to produce a new set of multiple-ping events for future pings.

Updated entries are made in the master status file in three functionally different ways. These are:

(a) Large single ping events,

(b) Single ping events which are linked with multiple ping events, and

(c) Large multiple ping events which are not linked to new peaks.

The procedures used for making each type of entry are described below.

A.3.1 Large Single Ping Events

If the log likelihood ratio of a single ping event exceeds the retention threshold, then a single ping entry to the master status file will usually be made. This single ping entry to the master status file will be blocked if the peak is linked in a very consistent way with an existing track. This blocking procedure is described under processing of linked events. On the first echo ranging cycle, only single ping entries are made. The expected range position is set equal to the single event range. The log of the track likelihood ratio is set equal to the single ping log likelihood ratio.

A.3.2 Linked Events and Large Multiple Ping Events Which Are Not Linked to New Peaks

When a single-ping event occurs in a range position in the vicinity of the expected position of a target track, the possibility of a "link" between the single-ping event and the multiple ping track entry arises. The linking process is the most essential part of the target track detection model since it is the vehicle for achieving ping-to-ping integration along a track.

The logic used to accomplish the linking of tracks with new peaks is shown in Fig. A2. Each block on the flow chart shown is numbered and the following numbered paragraphs provide functional descriptions of the corresponding flow chart blocks.

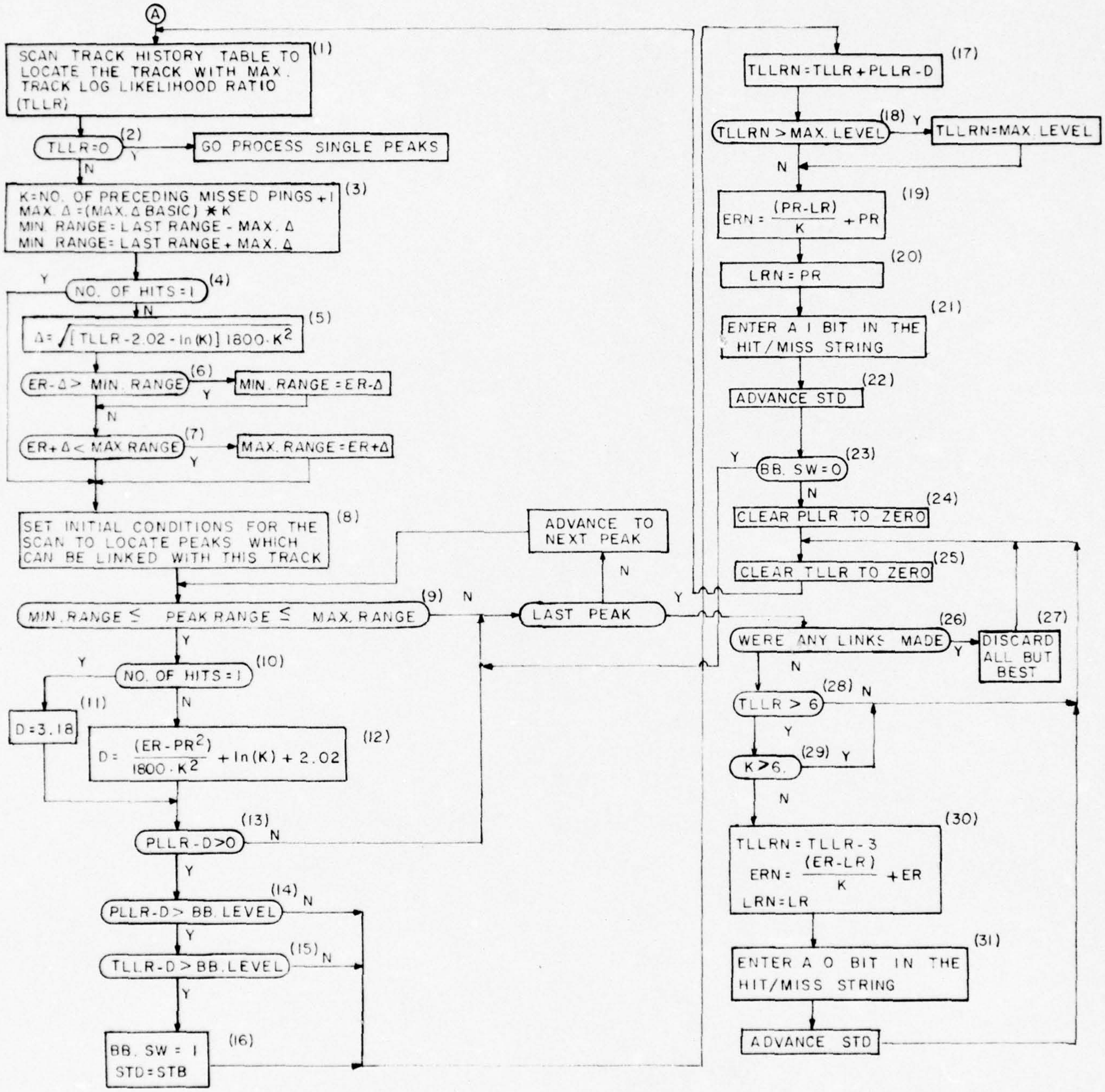

Fig. A-2 COMPUTER AIDED DETECTION FLOWCHART

(1) The history table is scanned to locate the history packet with the largest log likelihood ratio. Further processing described below will be directed toward linking the entry located with a peak in the current echo cycle or propagating the entry if no link is achieved.

(2) If the TLLR of the entry located in step (1) is zero, then all history packet entries have been processed and control is transferred to the routine to process single peaks as described in Section A.3.1 above. If the TLLR is zero, control passes on to step (3) below.

(3) This block of logic determines the minimum and maximum ranges applicable for linkage with a peak, based on the maximum range rate to be tracked. A quantity, K, is set equal to the number of preceding missed pings + 1. The maximum range change to be allowed, MAX. Δ, is calculated by multiplying a basic maximum range change by K. This process effectively opens the range gate when missed pings occur. The minimum and maximum ranges to be allowed are calculated from the last range measured by subtracting and adding MAX. Δ, respectively.

(4) A test is made to determine whether a range rate estimate is available. If the track under consideration has only one ping with a detection, then no range rate estimate is available and the logic proceeds to step (8). Otherwise it proceeds to step (5).

(5) This calculation determines the maximum deviation from the expected position that still permits a link to be made with a positive contribution to the peak log likelihood ratio from the link. In general, when a link is made the track log likelihood ratio is updated by an equation of the form,

$$(TLLR)_N = (TLLR)_O + PLLR - D(|R-ER|), \quad (A1)$$

where

$(TLLR)_N$ = the new track log likelihood ratio,

$(TLLR)_O$ = the old track log likelihood ratio,

PLLR = the single peak log likelihood ratio, and

$D(|R-ER|)$ = some deviation loss function of the difference between the expected range and the measured range.

The maximum deviation, Δ, is defined as the value such that

$$(TLLR)_O - D(\Delta) = 0 \quad . \quad (A2)$$

The algorithm used in computing D has been derived by assuming that the range distribution of signal peaks belonging to a track relative to the expected range is Gaussian with a mean of zero and a standard deviation of $(K\cdot\sigma)$. It is important to note that the algorithm does compensate for the increase in position ambiguity following missed pings. The actual algorithm used in computing D is

$$D = - \ell n \left(\frac{N}{K\cdot\sigma\sqrt{2\pi}}\right) + \frac{d^2}{2(K\sigma)^2} \quad , \quad (A3)$$

where N is the number of samples in a resolution interval at the signal processor output and d is the deviation from the expected position. By combining Eqs. (A1), (A2), and (A3) and solving for Δ one can obtain

$$\Delta = \sqrt{\left[TLLR + \ln\left(\frac{N}{\sigma\sqrt{2\pi}}\right) - \ln(K)\right] \cdot 2 \cdot (\sigma \cdot K)^2} \quad . \qquad (A4)$$

Analytic methods do not readily yield a value for σ, but inspection of typical target tracks indicates a value of 30 ms. The number of samples in a resolution interval, N, is 10 since the sampling rate used is 1000 Hz and the time resolution of the sonar signal processor is 10 ms. Insertion of these coefficients in Eq. (A4) yields

$$\Delta = \sqrt{[TLLR - 2.02 \ln(K)] \cdot 1800 \cdot K^2} \quad . \qquad (A5)$$

(6), (7) The purpose of this logic is to reset the range gate limits, if necessary, based on the maximum deviation that can be tolerated in the expected range.

(8) The initial conditions for the scan of peaks linked to a given track include:

(a) BB.SW=0, This switch is used to determine whether or not a "BRICK-BAT" link has been achieved as described below.

(b) Two storage control addresses are set, both initially pointing to the same address. STB will remain static, pointing to the first linked event package to be formed. STD will advance as successive event packages are formed, when more than one peak is linked to the given track.

(c) Necessary loop controls are set for a scan of all new peaks.

(9) Each peak is tested to be within the allowed range gate. When a peak is found within the allowed range gate, control passes to step (10).

(10) The next step after locating a peak within the allowed range gate is to determine the position deviation loss associated with the peak/track pair. If the track only has one previous "hit," then no range rate estimate is available and control passes to step (11). Otherwise control passes to step (12).

(11) When making the first link of a new track no range rate estimate is available, so the standard method for calculating the position deviation loss is not applicable. In this case the deviation loss applied must compensate for the ambiguity introduced in scanning the range gate to locate the peak. In the validation study the standard range gate is 240 ms. Since the resolution of the sonar is 10 ms there were 24 independent samples in the 240 ms gate. A deviation loss of $\ln$ (24), or 3.18, is applied to compensate for the link ambiguity.

(12) When a range rate estimate is available, Eq. (A3) is used to calculate the deviation loss D. As before, the coefficients N=10 and σ=30 are used. When these coefficients are inserted in Eq. (A3) one obtains.

$$D = \frac{d^2}{1800 \cdot K^2} + \ln(K) + 2.02 \quad . \tag{A6}$$

(13) From Eq. (A1) it can be seen that when a link is made the track log likelihood ratio will increase by an amount PLLR-D. If this quantity is negative the link is not made and control is returned to scan the next peak.

(14), (15), (16) At this point the decision to make a linked entry is complete. The purpose of the logic in steps (14), (15) and (16) is to classify the link as a "BRICK-BAT" link if both PLLR-D and TLLR-D exceed a fixed level, BB.LEVEL. The intent of the "BRICK-BAT" link is to assert that we are sure that the link is correct, and then to prevent any other interactions of the track package and the peak event. The reset of STD to STB will cause any linked event packages already formed with this track to be discarded.

(17) to (22) These steps carry out the necessary operations to form a linked event entry package in the new history file. The new event package is stored in the file at a position indicated by STD. The new track log likelihood ratio is calculated in step (17) using Eq. (A1). A limit is applied to the value allowed for the track log likelihood ratio in step (18). The new expected range, ERN, is calculated using the peak range, PR, and the last range, LR, obtained from the old track history packet. The

current peak range, PR, is entered in the new history packet as the new last range. A 1-bit is entered to indicate a hit in the hit/miss string. The storage control pointer, STD, is advanced in step (22).

(23) A test is made to determine whether "BRICK-BAT" conditions were met on the link just completed. If these conditions were not met control is returned to scan the next peak. If the conditions were met control proceeds to step (24).

(24) If a "BRICK-BAT" link was made the peak log likelihood ratio is cleared to prevent the peak from causing a single peak entry.

(25) The track log likelihood ratio in the old history packet is cleared to zero and control is returned to scan for the next track to be processed.

(26) Control reaches this point when all peaks have been tested for a link with a given track and a "BRICK-BAT" link has not been achieved. If no links were made, control passes to step (28) to test for packet propagation.

(27) If several links were made, only the best link is retained. The choice of the best link is based on the maximum TLLRN.

(28) Control reaches this point only if no links were made. If the track log likelihood ratio, TLLR, is less than 6, control is passed to step (25) and the track is dropped.

(29) If the number of successive missed pings exceeds 6, control passes to step (25) and the track is dropped.

(30), (31), (32) These steps carry out the necessary operations to propagate a track package from the old history file to the new history file. The track log likelihood ratio is reduced by 3 to reflect the effect of a missed ping. The expected range is extrapolated and a 0-bit is entered in the hit/miss string.

A.3.3 Track Redundancy Removal

The procedures described in Sections A.3.1 and A.3.2 above allow the possibility of generating redundant entries in the new history file. To eliminate redundancies, the new history file is scanned to locate entries for which both the last range, LR, and the expected range, ER are the same. When several of these entries have been located only the entry with the largest track log likelihood ratio is retained.

APPENDIX B. DERIVATION OF THE TRANSFORMATION FROM PEAK HEIGHT TO LOG LIKELIHOOD RATIO

The transformation from processor output peak amplitude to logarithm of likelihood ratio is the initial processing stage of the target tracking model. Model performance is directly related to the accuracy of the transformation. The validity of the log likelihood conversion algorithm is in turn dependent upon an accurate statistical description of processor output amplitude in a multi-ping signal-plus-noise environment. This Appendix presents two different statistical approximations to a multi-ping signal-plus noise distribution and documents their associated likelihood ratio transformations.

The likelihood ratio, L(x), associated with a processor output peak of amplitude x is given by

$$L(x) = \frac{P_s(x)}{P_n(x)} , \tag{B1}$$

where P_s is the probability density function of amplitude in a processor output exhibiting a signal-to-noise ratio S, and P_n is the probability density function of amplitude in noise-alone processor output.

One immediate problem arises, since the likelihood ratio L(x) defined above is dependent upon the average signal-to-noise ratio S assumed at the processor output. In order to implement the likelihood ratio conversion, the value of S must be fixed as a system parameter. Yet, the actual value of S cannot be known before the fact, and it may vary widely from one contact to another. One solution to this problem is to set S in the range of the minimum detectable signal-to-noise ratio at the output

of the processor. When this is done, likelihood ratio estimates computed according to Eq. (B1) will be inexact only for signal-to-noise ratios somewhat larger or smaller than S. This inexactness will not affect overall system performance because the smaller signals are presumably non-detectable at the processor output and the larger signals require less processing gain for detectability. Thus, when S is set at a marginally detectable signal-to-noise ratio, the tracking model is optimized to provide maximum processing gain for those signals that are only barely detectable. In the validation study, the region of minimal detectable signal-to-noise ratio at the output of the correlator is from 10 dB to 12 dB.

The next problem in deriving a log likelihood transformation is to determine the probability density functions P_s and P_n required for the computation of Eq. (B1). Two classes of probability density functions have been used to derive log likelihood ratio conversion algorithms. The first class of functions, discussed in Section B.1., arise from the assumption that the ping-to-ping signal-plus-noise statistics of correlator output amplitude are described by the envelope distribution of an ideal signal in Gaussian noise. The second class of functions, discussed in Section B.2., arise from the assumption that signal-plus-noise and noise-alone correlator output amplitude statistics are both described by Rayleigh distributions with different standard deviations.

B.1 LOG LIKELIHOOD RATIO OF IDEAL SIGNAL PLUS GAUSSIAN NOISE ENVELOPE

If the assumption is made that signal amplitude is constant from ping-to-ping, then echo returns following the

correlator are similar to short CW pulses of the same signal amplitude. The linear rectifier and averager following the correlator then act as an envelope detector for these pulses. Thus, the probability density functions required for evaluating Eq. (B1) are those which describe the envelope of a sine wave plus random noise. These density functions are given by S. O. Rice* as

$$P_S(X) = e^{-S/N} e^{\frac{-X^2}{2N}} I_o(\sqrt{2} \sqrt{S/N} \frac{X}{\sqrt{N}}) \quad ,$$

$$P_N(X) = \frac{X}{N} e^{-\frac{X^2}{2N}} \quad ,$$

where

X is the height of the envelope,

N is the average noise power,

S is the average signal power,

I_o is the modified Bessel function of order 0.

Using these two equations to compute likelihood ratio, Eq. (B1) yields

$$L(x) = e^{-S/N} I_o(\sqrt{2} \sqrt{S/N} \frac{X}{\sqrt{N}}) \quad .$$

---

*Rice, S. O., "Mathematical Analysis of Random Noise," Bell System Technical Journal, Vol. 24, 1945, p. 100.

As given by Rice, the output noise mean, μ, is $\sqrt{N\pi/2}$ and the output noise standard deviation σ is $\sqrt{N(2 - \pi/2)}$. If r denotes the envelope height in units of σ relative to μ, then

$$r = (X - \mu)/\sigma \ ,$$

or

$$X = \sigma \cdot r + \mu$$
$$= \sqrt{N(2 - \pi/2)} \cdot r + \sqrt{N\pi/2} \ .$$

Substitution for X in the equation for L(x) above yields

$$L(x) = e^{-S/N} I_o \left[\sqrt{S/N} \ (\sqrt{4-\pi} \cdot r + \sqrt{\pi})\right] \ . \tag{B2}$$

The conversion algorithm for correlator output amplitude samples X is then obtained by taking the natural logarithm of Eq. (B2) where X has been converted to r using measured values of μ and σ. This has been done in Fig. B-1, where the logarithm of Eq. (B2) has been plotted as a function of r for three values of S/N that yield average output signal-to-noise ratios of 10 dB, 11 dB and 12 dB.

The curves that appear in Fig. B-1 thus yield the fruitful result that the logarithm of Eq. (B2) is closely approximated by a straight line in the region of interest (i.e. values of r from 2 to 6 which correspond to output signal-to-noise ratios from 6 dB to 15.5 dB). The logarithm of likelihood ratio conversion of a measured output peak of amplitude X then reduces to

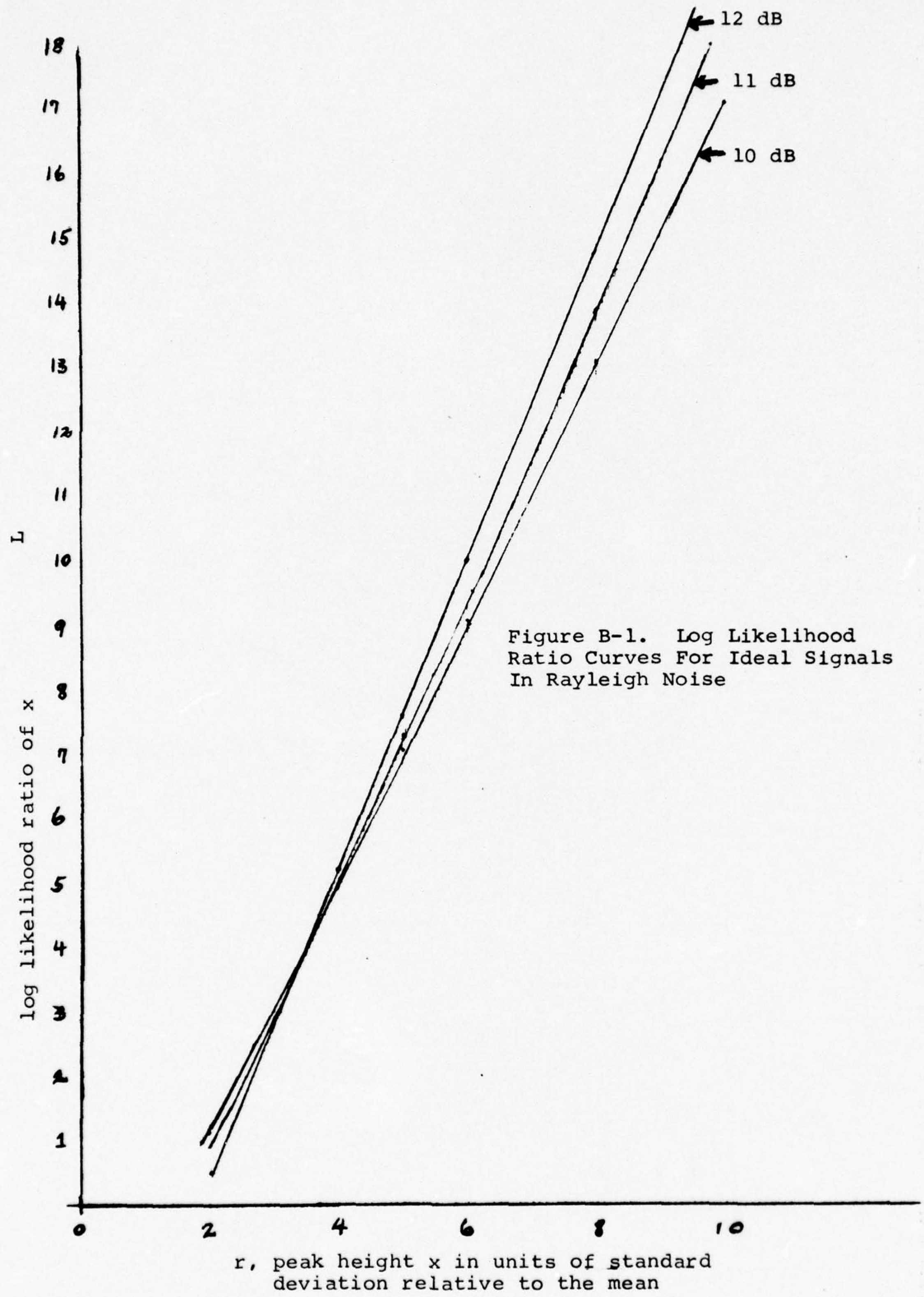

Figure B-1. Log Likelihood Ratio Curves For Ideal Signals In Rayleigh Noise

$$\ln L(x) = a \cdot r + b$$

$$= a\left(\frac{X-\mu}{\sigma}\right) + b$$

$$\ln L(x) = \frac{a}{\sigma} \cdot X + b - \frac{a\mu}{\sigma}\ , \qquad \text{(B3)}$$

where a and b are the slope and slope intercept values defining the appropriate straight line in Fig. B-1 and μ and σ are measured values of correlator output noise mean and standard deviation. For example, if the expected correlator output signal-to-noise ratio is to be set at 10 dB, then Fig. B-1 is examined and values of a = 2.0 and b = -2.86 are computed as the values determining the straight line marked 10 dB. In this case, then, the transformation from peak height X to logarithm of likelihood ratio $\ln L(x)$ is given by

$$\ln L(x) = 2.0 \frac{X}{\sigma} - 2.86 - \frac{2.0\mu}{\sigma} \quad .$$

In summary, the assumption that correlator output peak statistics are described on a multi-ping basis by the distribution of constant amplitude ideal signal plus Gaussian noise yields the easily implemented linear transformation in Eq. (B3) from peak height to logarithm of likelihood ratio.

B.2 LOG LIKELIHOOD RATIO OF RAYLEIGH SIGNAL PLUS RAYLEIGH NOISE

Although the log likelihood ratio conversion discussed in Section B.1 is simple to implement, the assumptions upon which it is based seem rather stringent. In particular, the assumption

that consecutive echo cycles contain ideal constant amplitude signals plus Gaussian noise is not confirmed by empirical data, which indicates that peak heights have a larger variation than that expected in an ideal case. For this reason, a "noisy" signal distribution was considered. This distribution is to be denoted as Rayleigh signal plus Rayleigh noise.

The Rayleigh probability density function, which is identical to the Rice noise-alone distribution, is given by

$$P(X) = \frac{X}{\alpha^2} e^{-\frac{X^2}{2\alpha^a}} .$$

The signal-plus-noise hypothesis investigated in this section assumes that both signal-plus-noise and noise-alone peaks are distributed by Rayleigh density functions P_S and P_N with different parameters α_S and α_N. Hence,

$$P_S(X) = \frac{X}{\alpha_S} e^{-\frac{X^2}{2\alpha^2{}_s}} ,$$

and

$$P_{\sqcup}(S) = \frac{X}{\alpha_N} e^{-\frac{X^2}{2\alpha^2{}_N}} .$$

Under the assumption of the above density factors, the likelihood ratio measurement L(X) is given by

$$L(X) = \frac{\alpha_{\sqcup}^2}{\alpha_S^2} e^{X^2\left(\frac{1}{2\alpha_N^2} - \frac{1}{2\alpha_S^2}\right)} .$$

The mean values μ_s and μ_n and standard deviation values σ_s and σ_n of the signal and noise distributions are given by

$$\mu_s = \alpha_s \sqrt{\pi/2} \quad ,$$

$$\mu_n = \alpha_n \sqrt{\pi/2} \quad ,$$

$$\sigma_s = \alpha_s \sqrt{2-\pi/2} \quad ,$$

$$\sigma_n = \alpha_n \sqrt{2-\pi/2} \quad .$$

Consequently, amplitude values X, when measured in units r of noise standard deviation σ_n relative to the noise mean μ_n are given by

$$X = \sigma_n r + \mu_n$$

$$= \alpha_n \sqrt{2-\pi/2} \cdot r + \alpha_n \sqrt{\pi/2} \quad .$$

Substituting the above for X in the previous expression for L(X) the following is obtained:

$$L(X) = \frac{\alpha_n^2}{\alpha_s^2} e^{\alpha_n^{\,2}[k_1 r+k_2]^2 \left(\frac{1}{2\alpha_n^{\,2}} \; \frac{1}{2\alpha_s^{\,2}}\right)}$$

$$\approx \frac{\alpha_n^2}{\alpha_s^2} e^{[k_1 r+k_2]^2 (\frac{1}{2} - \frac{d_n^{\,2}}{2\alpha_s^{\,2}}} \quad ,$$

where $k_1 = \sqrt{2-\pi/2}$ and $k_2 = \sqrt{\pi/2}$.

Finally, taking log likelihood ratio, we obtain

$$\ln L(X) = \ln(\frac{\alpha_n^2}{\alpha_s^2}) + (\frac{1}{2} - \frac{\alpha_n^2}{2\alpha_s^2}) [k_1 r + k_2]^2 \quad . \tag{B4}$$

The relation between $\frac{\alpha_n}{\alpha_s}$ and correlator output signal-to-noise ratio is easily obtained. For example, since the tracking model is to be optimized for 10 dB, it is required that

$$\begin{aligned} 10 &= 20 \log (\frac{\mu_s - \mu_n}{\sigma}) \\ &= 20 \log (\frac{\alpha_s k_2 - \alpha_n k_2}{\alpha_n k_1}) \\ &= 20 \log (\frac{\alpha_s}{\alpha_n} \frac{k_2}{k_1} - \frac{k_2}{k_1}) \quad . \end{aligned}$$

Consequently,

$$\frac{\alpha_s}{\alpha_n} = \frac{k_1}{k_2} \sqrt{10} + 1 = 2.65 \text{ and } \frac{\alpha_n}{\alpha_s} = 0.377 \quad .$$

Thus, to optimize Eq. (B4) for a 10 dB correlator output signal-to-noise ratio, the value 0.377 is substituted in Eq. (B4) for $\frac{\alpha_n}{\alpha_s}$ and the following conversion algorithm results:

$$\ln L(X) = \ln(0.377)^2 + (\tfrac{1}{2} - 0.377^2)\,[k_1 r + k_2]^2$$

$$= -1.275 + (0.706)r + (0.185)r^2 \quad .$$

Equation (B4) has been evaluated for values of $\frac{\alpha_n}{\alpha_s}$ corresponding to correlator output signal-to-noise ratios of 10 dB and 11.5 dB. The resulting curves, in which $\ln L(X)$ is plotted as a function of r, appear in Fig. B-2. The quadratic transformation indicated in Eq. (B4) may be simplified further by computing a least mean square linear fit in the region of interest to the curves in Fig. B-2. This has been done for the 10 dB curve and the resulting linear fit is indicated by the dotted line. The slope and slope intercept values a and b yield a simplified log likelihood transformation given by

$$\ln L(X) = 2.45r - 5.2 \quad .$$

In summary, the assumption of a Rayleigh signal plus Rayleigh noise distribution yields a quadratic transformation from peak height X to log likelihood ratio that is quite practical to implement and which further may be approximated by a linear transformation in the range of interesting signal-to-noise ratio. This form of log likelihood ratio transform is being used in the validation study.

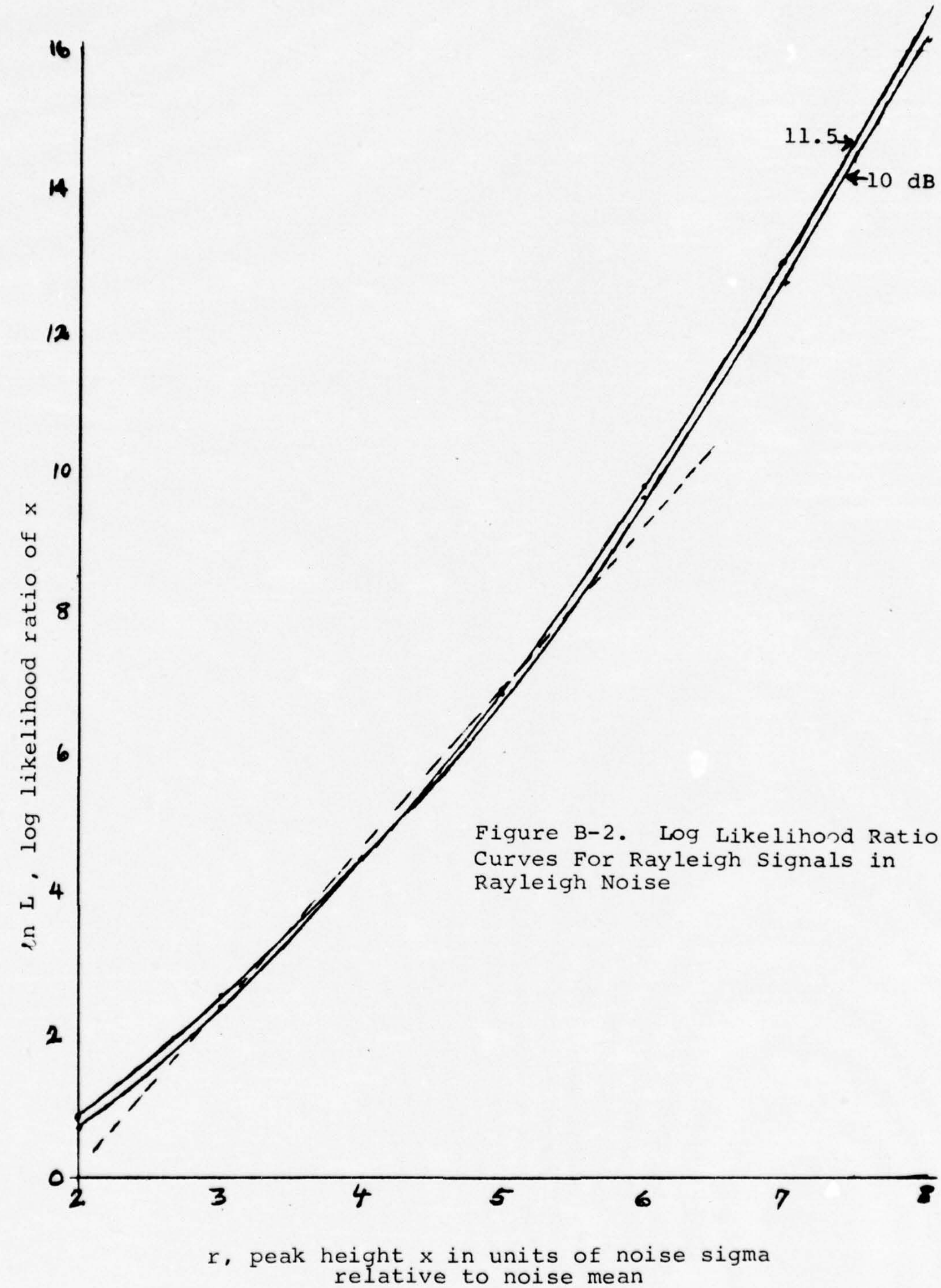

Figure B-2. Log Likelihood Ratio Curves For Rayleigh Signals in Rayleigh Noise

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APPENDIX C. A SIMPLIFIED MODEL FOR PREDICTING CAD PERFORMANCE

To provide some insight into the expected performance of the computer-aided detection model, it is useful to make some simplifying assumptions about the data. The assumptions to be made are:

(1) The peaks of a signal track will be assumed to be equal to the same value on each ping.

(2) The deviation of the peak position from the expected position will be assumed to have a standard deviation which is equal to the standard deviation used in calculating the position deviation loss.

When a peak is linked to a track the track log likelihood ratio is updated by an equation of the form,

$$TLLRN = TLLRO + PLLR - D(d) \quad , \tag{C1}$$

where

TLLRN = the new track log likelihood ratio,
TLLRO = the old track log likelihood ratio,
PLLR = the log likelihood ratio of the peak,
D(d) = a subtractive loss which is a function of the deviation of the peak position from the expected position.

The subtractive loss function, D, compensates for the ambiguity which must be allowed to make the link. The evaluation of the D function depends on whether or not a range rate estimate is available. In the first link of a new track no range rate estimate is available, and the D function is evaluated as the log of the number of independent samples in the allowed link range

gate. In the validation study the allowed link range gate is 240 ms and the resolution of the sonar is $\ell n(24)=3.18$. When a range rate estimate is available the D function is evaluated as

$$D(d) = \ell n(\frac{\sigma\sqrt{2\pi}}{N} + 1) + \frac{d^2}{2\sigma^2} \quad , \qquad (C2)$$

where

σ = the standard deviation assumed for the difference between the expected range and measured range for signals.*

N = the number of independent samples in one resolution cell at the sonar output,

d = the difference between the expected range and the measured range.

Using assumption (2) Eq. (C2) reduces to

$$D = \ell n(\frac{\sigma\sqrt{2\pi}}{N} + 1) + \frac{1}{2} \quad . \qquad (C3)$$

Now it is possible to write an expression giving the expected track log likelihood ratio, TLLR, as a function of the number of pings, K, and the peak log likelihood ratio, PLLR:

*In the implementation of the CAD model the approximation $\frac{\sigma\sqrt{2\pi}}{N} + 1 \simeq \frac{\sigma\sqrt{2\pi}}{N}$ has been used. The more exact form has been included here to allow parameter variations over a range such that the approximation is not valid.

If K = 1

$$TLLR(PLLR,K) = PLLR$$

If K ≥ 2

$$TLLR(PLLR,K) = K \cdot PLLR - 3.18 - (K-2)\ (\ell n(\frac{\sigma\sqrt{2\pi}}{N} + 1) + \frac{1}{2})$$

$$= K \cdot (PLLR - \ell n(\frac{\sigma\sqrt{2\pi}}{N} + 1) - \frac{1}{2}) + 2 \cdot \ell n(\frac{\sigma\sqrt{2\pi}}{N} + 1) - 2.18 \quad .$$

From Eq. (C4) one can see that the expected track log likelihood ratio is a linear function of the number of pings, K, when K is greater than one. It is important to note that the expected track log likelihood will increase with K only if the peak log likelihood ratio is greater than $\ell n(\frac{\sigma\sqrt{2\pi}}{N} + 1) + \frac{1}{2}$. This implies that the tracking ambiguity establishes a floor on PLLR such that signals of lower level cannot be detected even with an infinite observation time.

In the validation study the values used for σ and N were 30 and 10 ms respectively. Figure C-1 shows curves of expected track log likelihood ratio vs. number of pings with the value of average peak log likelihood ratio parameterized. As shown in Appendix B the peak log likelihood is directly related to signal-to-noise ratio. The values of signal-to-noise ratio equivalent to the log likelihood values are also given in Fig. C-1. Note that the range of signal-to-noise ratio from 10 dB to 13 dB covers the "interesting" range of operation. That is, signals below 10 dB cannot be detected and signals above 13 dB will be detected very easily.

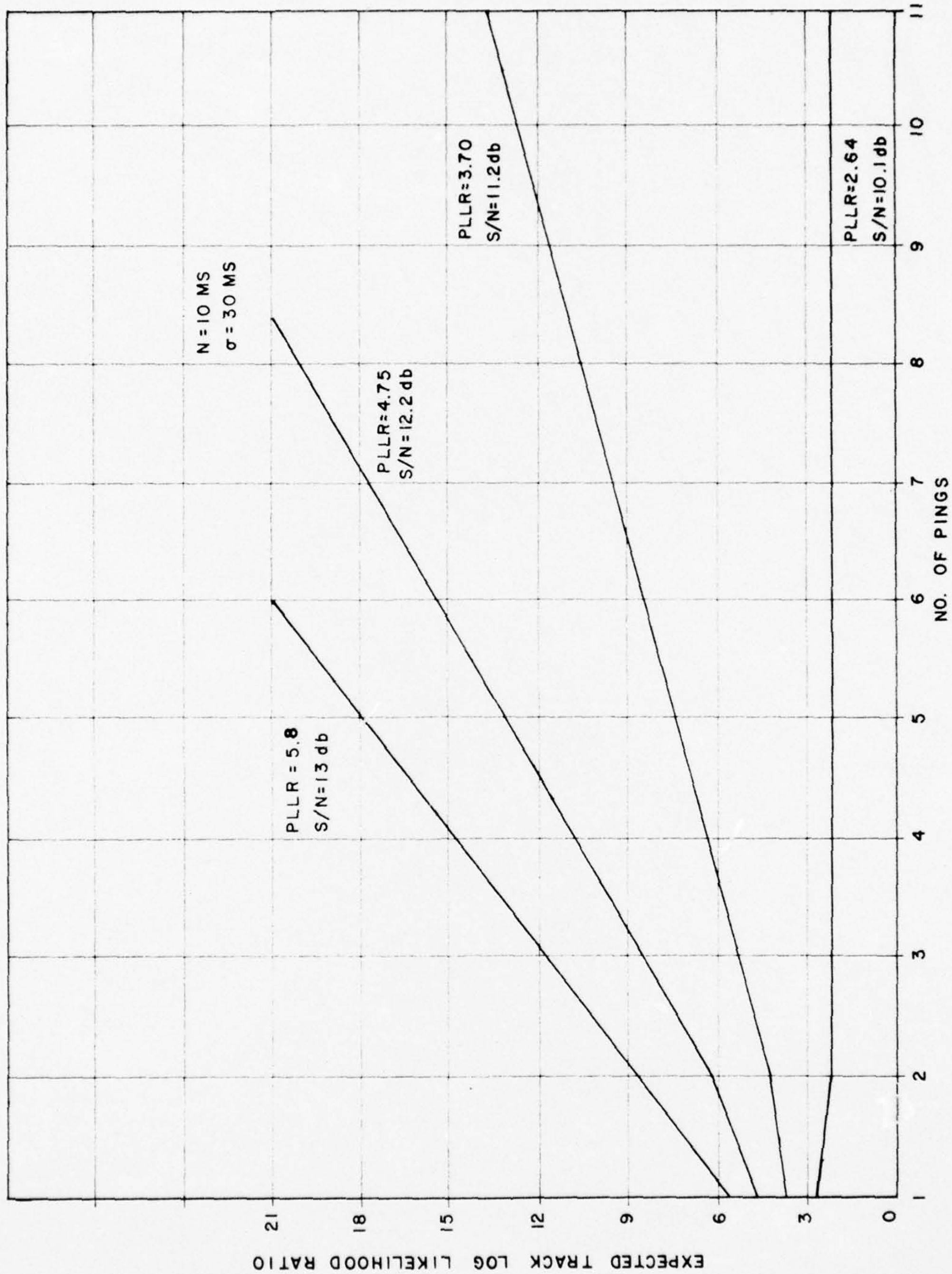

Fig. C-1 EXPECTED TRACK LOG LIKELIHOOD RATIO vs. NO. OF PINGS

Figure C-2 shows a set of curves which are functionally identical to those of Fig. C-1 with $\sigma = 10$ ms. The lowest detectable level is decreased to 9.1 dB by the improved track consistency.

Next, it is interesting to consider how the false alarm rate curves may be expected to change as the standard deviation assumed for the difference between the expected range and measured range, σ, is changed. Changing this parameter has an impact on the model as indicated in Eq. (C2). The intent of the deviation loss function, D, is to provide a normalized system such that we may vary σ without changing the false alarm characteristics. A large set of noise data was processed through the model using values for σ of 30 ms and 10 ms. The resulting false alarm rate curves are shown in Figs. C-3 and C-4. The close agreement between the two curves indicates that the deviation loss function as given in Eq. (C2) does achieve a normalized system.

By combining the curves of Fig. C-1 with the false alarm rate curve given in Fig. C-3 it is possible to obtain a set of curves of required signal-to-noise ratio vs. false alarm rate with the number of pings parameterized. These curves are shown in Fig. C-5. These curves may be compared directly with the measured curves obtained using sea data (Figs. 10 through 14 in Section 3.3). Considering the simplicity of the model used to obtain the curve of Fig. C-5, the agreement with the measured results is excellent, with differences between the two sets of curves on the order of $\frac{1}{2}$ dB.

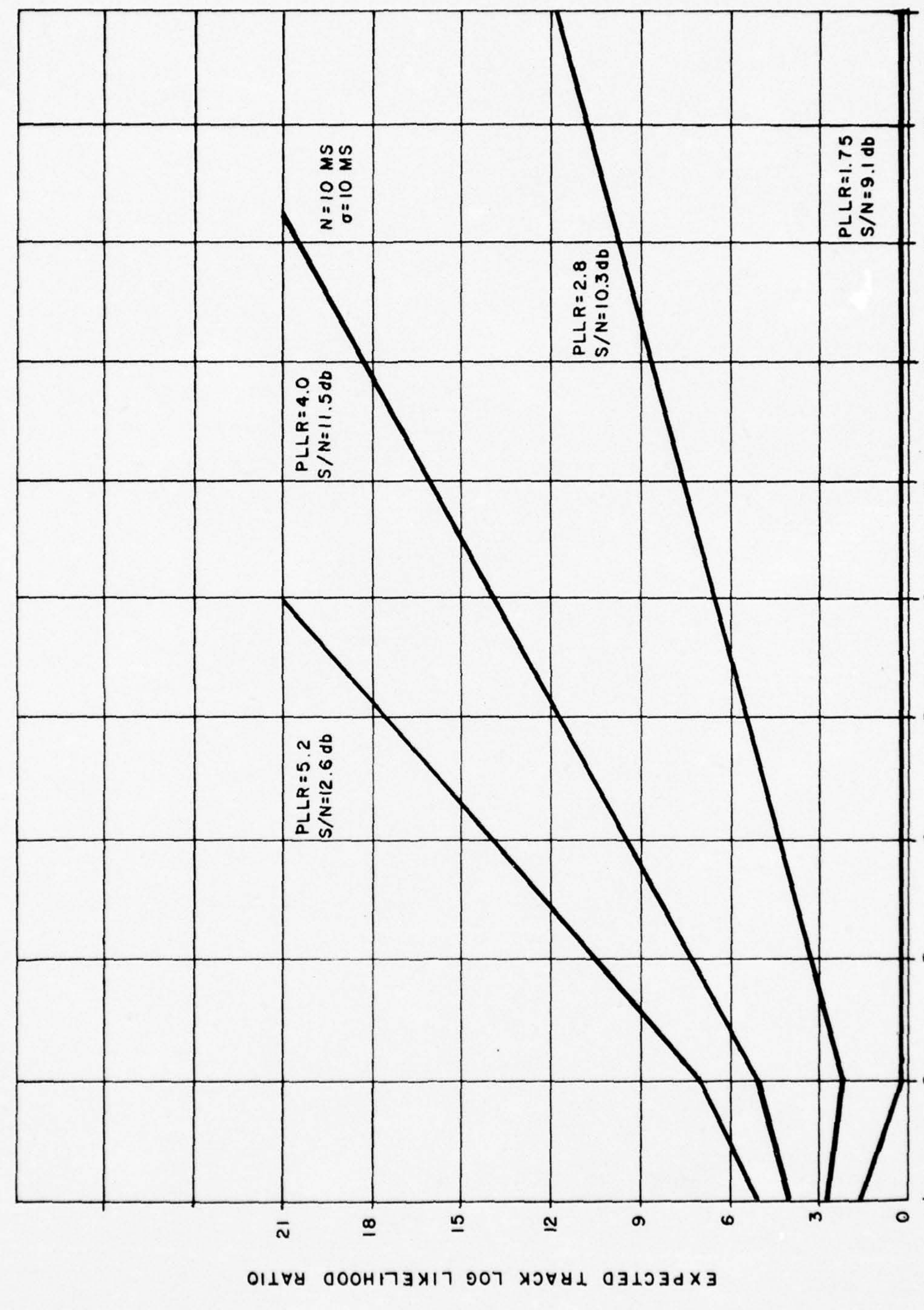

Fig. C-2 EXPECTED TRACK LOG LIKELIHOOD RATIO vs NO. OF PINGS

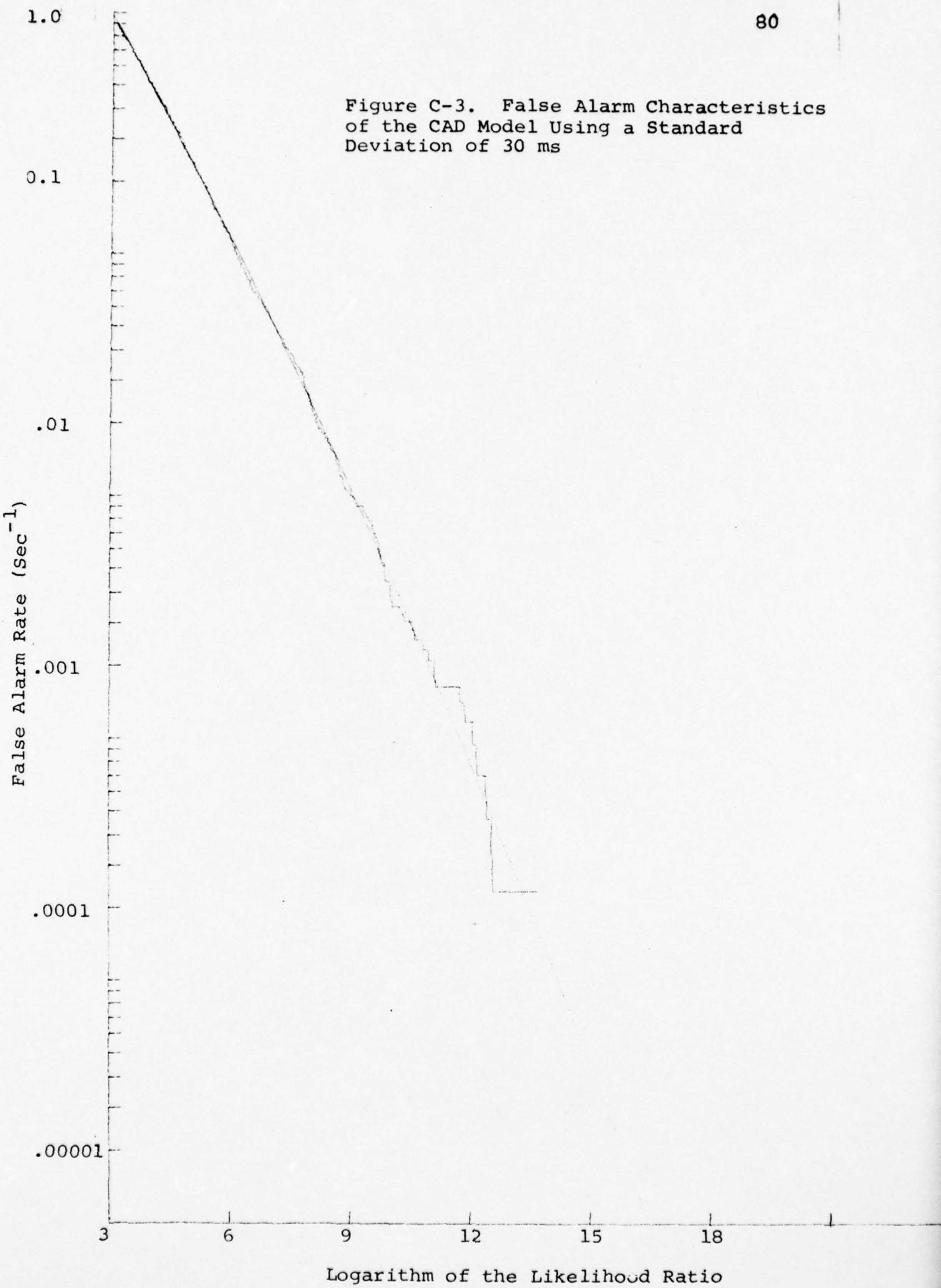

Figure C-3. False Alarm Characteristics of the CAD Model Using a Standard Deviation of 30 ms

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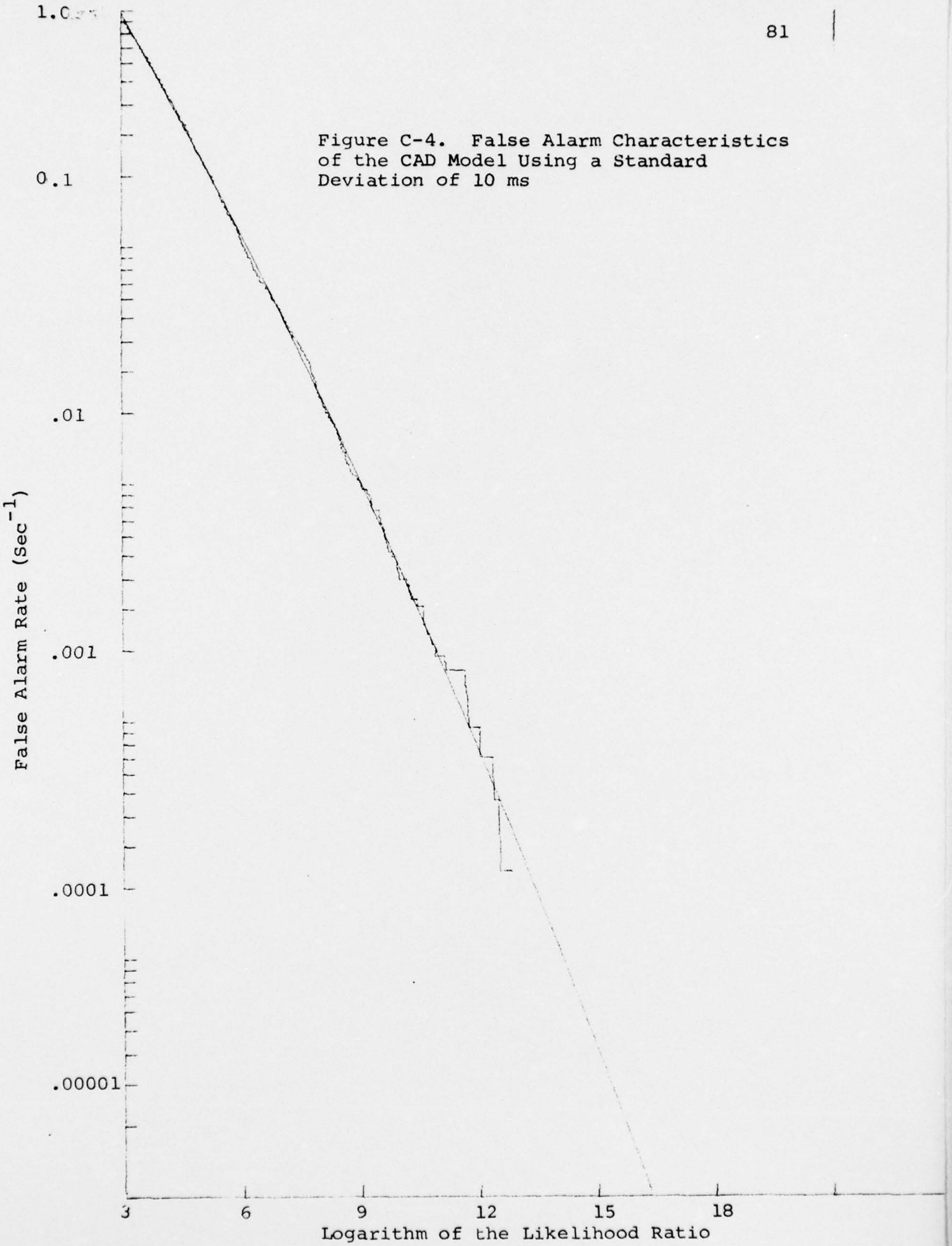

Figure C-4. False Alarm Characteristics of the CAD Model Using a Standard Deviation of 10 ms

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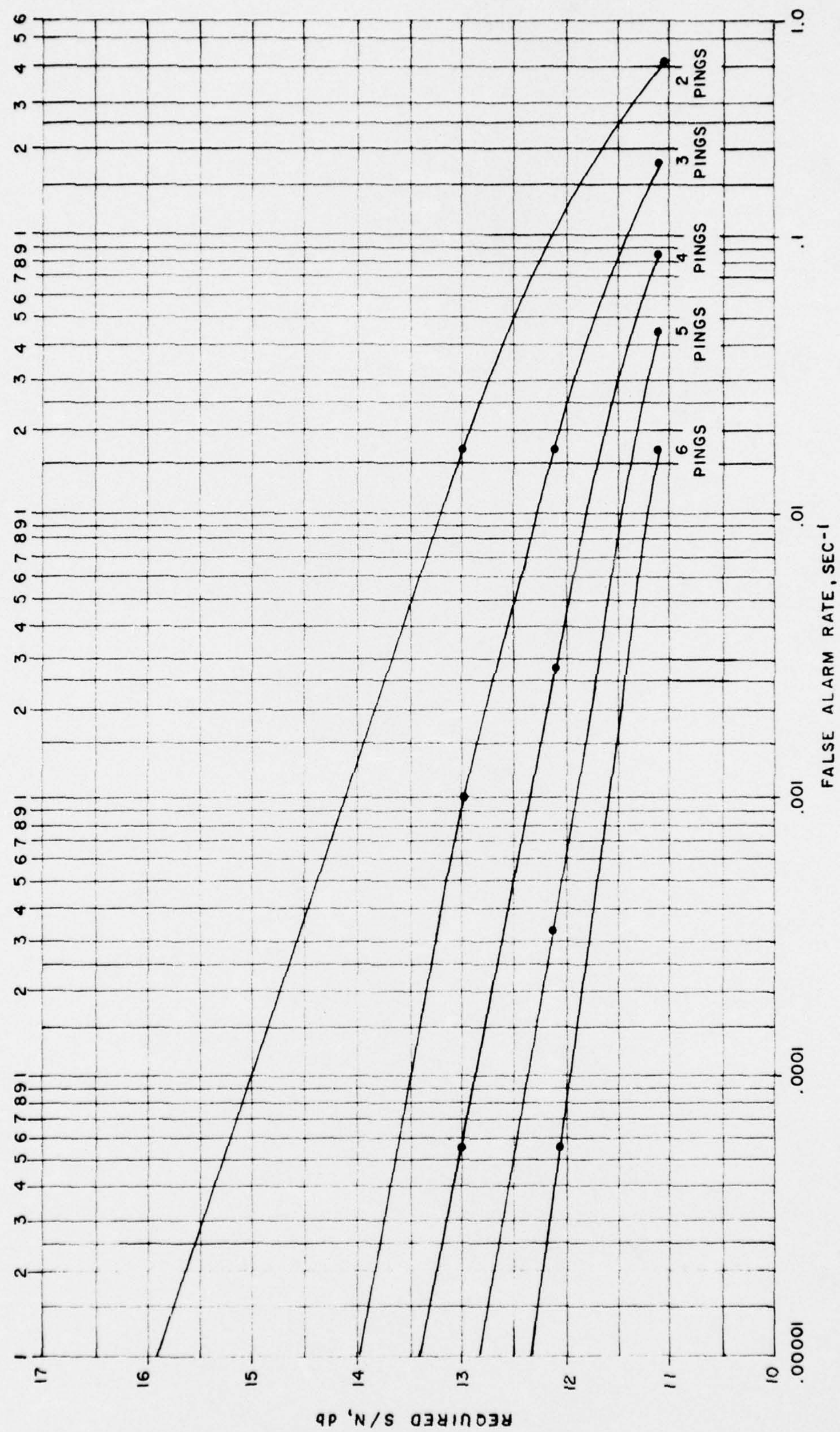

Fig. C-5 SUMMARY OF SIMPLIFIED MODEL PERFORMANCE

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APPENDIX D. SOME EXAMPLE DISPLAYS

In this appendix several sets of displays obtained from sea data are presented to give a visual comparison of displays with and without CAD processing. Each set consists of four displays which will be referred to as types A, B, C, and D. Each type of display is described below.

Type A Display

The type A display provides signal and noise information without CAD processing. The display is formatted with one line of print for each ping. Horizontal position within the display is proportional to time within the ping cycle relative to the start of a range gate. The digits printed indicate dB in excess of some given signal-to-noise ratio value provided in the header at the top of the display. The header also provides the start time of the range gate relative to the transmit pulse in ms, and the range bin width in ms.

Type B Display

The type B display provides a tabular representation of information about the target track selected by the automatic track localization process. The first column is the ping number. The second column is arrival time, relative to transmit time, of the peak selected in seconds. The third column is the signal-to-noise ratio of the peak in dB. The fourth column is the deviation of the peak position from a least mean square quadratic fit to the values in column two, in ms. At the bottom of the table the standard deviation of the values in column four is given to provide a measure of track position consistency.

Type C Display

The type C display is functionally like the type A display, but only the points of the selected target track are given.

Type D Display

The type D display is similar in format to display types A and C. The results displayed are the signal plus noise output of the CAD model. A limit of 7 exists for this display as a result of the limit on maximum log likelihood ratio imposed in the CAD model. Only pings with a signal present are displayed. Pings with a missed signal will be blank even though the CAD model has not discarded the track. A comparison of display types A and D provides a good visual indication of the impact of the CAD model.

```
ST3,R1,C3****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB, RANGE START IS 33018 SAMPLES,  113 SAMPLES PER RANGE BIN

    12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
 1       0          5 0                          0            2                              0
 2      0           8   2              0             2                                 2   1               0
 3          2          21        5
 4                                                          2      0                1
 5       4          2    2      2             0  0         2                                    2
 6                  0 1                 0   0     1                0                      1
 7                 4  0           0                           4
 8    1        1    0                           0    0                   0    1        5                 1
 9                               1                                                                  4 0
10       2         2                  0      1     0                5  1
11          2       07    2                                     0    2          0            0 0   0     2
12                                                                              0                 0
13         0                                    3                         2    1       0               1
14          0   03   0    0                                   1      10                  0       1
15   0               3         1         201      2                    0                     1         0
16                                                             5                            1   3       1
17       4   3       9       0        2  4    0            3                2   2
18       0      0    7  0          4       0         4     0
19         2 2       9    3      02     1  0     0     2  0    1                   0
20     2   0          9                                             1    1                                  1
```

Fig. D-A1

PING NO.	SEC.	(S/N)	DEV
1	34.900	16.46	3.4
2	34.911	18.85	-4.5
3			
4			
5	34.915	13.48	.8
6	34.919	11.12	-0.1
7	34.919	10.90	3.0
8	34.925	11.37	.1
9			
10			
11	34.949	17.91	-14.6
12			
13	34.927	10.41	13.5
14	34.941	10.82	2.6
15	34.947	14.43	-0.4
16			
17	34.960	19.97	-7.3
18	34.947	18.02	8.8
19	34.967	21.26	-8.2
20	34.959	21.56	2.8

SIGMA = 6.813

Fig. D-B1

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```
                         ST3,R1,C3****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB, RANGE START IS 33018 SAMPLES,  113 SAMPLES PER RANGE BIN

    123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345
   1                 5
   2                 8
   3
   4
   5                 2
   6                 0
   7                 0
   8                 0
   9
  10
  11                  7
  12
  13
  14                  0
  15                  3
  16
  17                  9
  18                  7
  19                  9
  20                  9
```

Fig. D-C1

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```
                    ST3,R1,C3****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE OR IN EXCESS OF  16 DB, RANGE START IS 33018 SAMPLES,  113 SAMPLES PER RANGE BIN

    12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
 1                  0
 2                  6
 3                               0
 4
 5                  5           0
 6                  5
 7                  6
 8                  6                                                                  0
 9
10                                                                  0
11                   7                                               2
12
13                  7
14                   7
15                   7
16                                                               0
17                   7
18                   7
19        1          7            0
20                   7
```

Fig. D-D1

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PERMIT FULLY LEGIBLE PRODUCTION

S13.R1.C2****PHASE I DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF 11 DB. RANGE START IS 36753 SAMPLES. 88 SAMPLES PER RANGE BIN

```
   12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
 1                8         1                   3 2       0    2     23             5
 2       0    1   2 3         2  0       2 0              0                                 0
 3     2           3                                                                            4
 4                       0             0             0              1       5                 1
 5      0          9     2   1                                      1            1       0         0
 6        1  1 3    9           3 2            1    0         0 3                           0        0
 7    2     0       1        0              0   0        0      4      0                         00       0
 8       1 4 0      7 1       11     1           4   22      0  0      2        0       0    2       1
 9                   3        3              41                                 1               0
10     1                 5       0       4 2              1      1    0                              3
11                   7                    1  5      3        1                      1 03     1      2
12     3        2    9   1                     2        2   2    33      2  0    3             3       0
13          0         7        2                                 0                 0             11  51
14                    0                           0      1          1  2   0                      1
15                    5                             11   1                  1        0  0       0        03
16           3   1  0       0  03           4      7     3        2      1           0   3  0
17               0     4 1     0      0        3    2                1                     2     4
18                     5         1 2                  0            00   1 0          0    1    0       22
19       3    0        9               1               4   1                   0   3      0       2       0
```

Fig. D-A2

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PING NO.	SEC.	(S/N)	DEV
1	38.131	19.38	-6.2
2	38.145	12.58	9.5
3	38.188	14.20	-4.3
4			
5	38.238	20.41	2.3
6	38.267	22.45	.8
7	38.290	12.04	4.8
8	38.323	17.80	-7.8
9	38.346	13.67	1.0
10	38.385	10.30	-12.7
11	38.379	17.50	18.0
12	38.420	20.26	1.2
13	38.452	18.25	-7.1
14	38.485	21.54	3.0
15	38.499	14.55	-8.5
16			
17	38.528	14.81	5.9
18	38.551	19.02	3.8
19	38.574	21.35	-3.9

SIGMA = 7.370

Fig. D-B2

```
                    ST3,R3,C2****PHASE I DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB, RANGE START IS 36753 SAMPLES,   88 SAMPLES PER RANGE BIN

    12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
  1               8
  2               7
  3                 3
  4
  5                 9
  6                  4
  7                  1
  8                  7
  9                    3
 10
 11                    7
 12                    4
 13                     7
 14                     4
 15                     4
 16
 17                      1
 18                      1
 19                      1
```

Fig. D-C2

```
                  SI3.R3.C2****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  16 DB. RANGE START IS 36753 SAMPLES.   88 SAMPLES PER RANGE BIN

    12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
  1                3                                                                  0
  2                4
  3                 6
  4                 6                                                         0                 0
  5                 7
  6                  7
  7                  7                                           1
  8                  7
  9                   7        0
 10                   7   0
 11                   7                       1
 12                   7                                            0
 13                    7                                                                               0
 14                    7
 15                    7
 16                                                 2     0
 17                     7
 18                     7
 19                     7
```

Fig. D-D2

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ST3,R9,C1****PHASE I DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF 11 DB, RANGE START IS 34561 SAMPLES. 90 SAMPLES PER RANGE BIN

```
   1234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
 1                  8            20   1   4             0     0 2     1         3             2     21
 2  1                 1                          0 1                                                         0 1
 3    11            9                      1                           0              1    1          1 1
 4                  7                   1                    3   1                                 0                  3
 5  2          1    9      5    3             1     1                   12                  1     2      4    2
 6 0                9                     1        0         2 4      6     0 2                                        2
 7           1      9              0                   3                        2                         0   0     1
 8                  5                                                                            2
 9     0            6             1              0       2                             0 0
10                  9        0                               3         0         1             1
11                  64            5              1      0  2        01                                          0   2
12                  62         5                        0                   3        1      0            0
13  1               8          0   1  4             2               4         2
14         1        2                          2                     0                    2
15       3                                     1 0      0                       0    0 1              0      4
16                   5 0           3   2             1      1                                 0    1      3     0          1
17                  8           0                  0    5  7                25     1                                        5
18           1      32 2                                0                 0     00            2   00          0            2
19 0                 2    1 1              0   4        3     01         5  0    1      0 2       6   1      0      4
20                   9            0     0     1                                                        1     0        3     2
```

Fig. D-A3

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PING NO.	SEC.	(S/N)	DEV
1	35.859	18.70	8.5
2			
3	35.868	19.64	4.0
4	35.901	17.93	-26.6
5	35.874	23.80	2.8
6	35.880	23.64	-0.7
7	35.871	22.72	10.9
8	35.874	16.19	10.6
9	35.900	16.66	-12.7
10	35.886	21.53	4.1
11	35.896	17.36	-3.0
12	35.892	17.12	3.9
13	35.897	19.21	2.0
14	35.910	12.75	-7.9
15			
16	35.913	16.31	-4.5
17	35.900	18.81	11.8
18	35.909	13.63	6.2
19	35.924	12.57	-5.4
20	35.926	21.79	-3.9

SIGMA = 9.270

Fig. D-B3

```
                ST3.R9.C1****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB. RANGE START IS 34561 SAMPLES.    90 SAMPLES PER RANGE BIN

     12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
   1                       8
   2
   3                       9
   4                       7
   5                       9
   6                       9
   7                       9
   8                       5
   9                       6
  10                       9
  11                       6
  12                       6
  13                       8
  14                       2
  15
  16                        5
  17                       8
  18                       3
  19                        2
  20                        9
```

Fig. D-C3

```
               ST3,R9,C1****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  16 DB, RANGE START IS 34561 SAMPLES,   90 SAMPLES PER RANGE BIN

      12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
   1              3
   2
   3              7
   4              7
   5              7     0
   6              7                                    0     2
   7              7
   8              7
   9              7
  10              7
  11              7           0
  12              71       0
  13              7        0
  14              7
  15
  16               7
  17              7                             0  3               0                                      1
  18              73                            0
  19               7                             1             0                     2
  20               7                                                                               1
```

Fig. D-D3

CONFIDENTIAL

```
                    S[3.R24.C]****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB. RANGE START IS 3371 SAMPLES.

     1234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456
  1             1                          1             0
  2           0          4          2        3
  3              0       4                 0
  4           1          7     2                  1      3
  5         0    1       1      1    03    30
  6         0   1      5 0           0
  7               3  4        1          1
  8           0    41         3    1     3  22   2          16
  9    1 0    1        2          32  2
 10         1            1  3        0  2  2
 11         1       40   2                 5    1
 12 0              4            3         20         2    1
 13                        3            0     2
 14         0  1  9               1      3    1
 15      3        9          1     3        2 2
 16 2   2   4     4        3     1  01
 17             5          3              1
 18              1 0        11
 19    0      1       2                 3      3
 20 4     1   4     2      1  1  26          1      1 1
```

Fig. D-A4

PING NO.	SEC.	(S/N)	DEV
1			
2	35.468	14.62	10.6
3	35.831	20.21	-17.2
4	35.731	17.86	17.4
5	35.691	10.22	-8.6
6	35.619	16.50	-3.6
7	35.535	14.87	13.3
8	35.496	16.10	-15.[illegible]
9			.
10			
11	35.253	11.08	19.0
12	35.247	14.92	-45.8
13			
14	35.045	21.22	12.8
15	34.964	26.64	21.0
16	34.907	15.39	4.6
17	34.832	16.67	5.5
18			
19	34.692	12.42	-4.6
20	34.620	14.44	-8.7

SIGMA = 17.144

Fig. D-B4

CONFIDENTIAL


```
                    S13.R24.C1****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB. RANGE START IS 33716 SAMPLES.   [illegible] SAMPLES PER RANGE [illegible]

     12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
  1
  2                             4
  3                             9
  4                            7
  5
  6                          6
  7                         4
  8
  9
 10
 11                     6
 12                     4
 13
 14                  9
 15                 9
 16                 4
 17                6
 18
 19              1
 20              4
```

Fig. D-C4

CONFIDENTIAL

```
                    ST3.324.C1****PHASE I DATA REDUCTION      PART ONE

DIGITS BELOW ARE DB IN EXCESS OF  16 DB. RANGE START IS 33716 SAMPLES.   48 SAMPLES PER RANGE BIN

   1234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123
 1
 2
 3                      0
 4                     7
 5
 6                    7
 7                   7
 8               1
 9
10
11                5                          0
12                5                         1
13                                          0
14              7                           2
15             7                           3
16     0       7
17            7
18
19          7                           0                                                      2
20          7                                                                  1
```

Fig. D-D4

ST3.R24.C1****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF 11 DB. RANGE START IS 32143 SAMPLES. 96 SAMPLES PER RANGE BIN

```
   1234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
 1            4         3               1         0    1               0               0
 2               1      7                   0     30   3     2                            2     5
 3                     9       0                       0  1            0   1 1         3
 4           0         0   0  0     0          0     0          11      22      4     0   0   2
 5              3     5     0                        3        3      1     3        0  0     0
 6                   4    0             3   0         0     1    0 0   3 0    1      0
 7                            4     1      0           1        0  3              13    1
 8               1  3  00      0     1 1       1   0             0             1     12
 9       3           1  1                 2   0   4  2  2   0      0         0
10               0       0 1       1 2 0    1    2     2                          1    30
11             2 64         1    0   0         0  0    0          2                          0
12            0  3  1       2  0                2    6  33    3    02              1  20
13  3      1          4    1       0           1    1       1         4 4   5      0    22    0
14  1     1  1 02     0    0                                   2    0        1    0 4  0        0
15      2       1     2   4         1       0  1    2                 0               00  3
16     4          0 0          0  1        12   0           2                 0      0 3       5
17    2        3         1     12       3  0   1     0   1      0            3
18        1       3   15    1                       71          0        1   0            4    2
19  0     0  2                    0  1        0       1 3    3  0  0  00       0          3  3
20  1   0     2  3        0                            2 5            0      0      1 1       0
```

Fig. D-A5

PING NO.	SEC.	(S/N)	DEV
1	34.418	13.77	15.2
2	34.365	18.07	5.5
3	34.282	20.80	~~25.4~~
4	34.268	11.11	-24.0
5	34.195	16.21	-14.7
6	34.150	14.71	~~-33.4~~
7	34.070	10.19	-18.2
8	33.945	14.30	2.1
9	33.921	~~9.41~~	~~1.0~~
10			
11	33.771	16.78	19.8
12	33.693	13.98	31.7
13	33.638	10.48	29.3
14	33.592	12.73	-0.5
15	33.534	11.90	-14.6
16			
17	33.401	13.61	-11.8
18			
19	33.256	12.75	-3.4
20			

SIGMA = 18.2[illegible]

Fig. D-B5

```
                    ST3.R24.C1****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB. RANGE START IS 32143 SAMPLES.   96 SAMPLES PER RANGE BIN
    1234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567
  1                        3
  2                        7
  3                       9
  4                       0
  5                      5
  6                     4
  7
  8                    1
  9
 10
 11                 6
 12                 3
 13
 14                2
 15               1
 16
 17              3
 18
 19            2
 20
```

Fig. D-C5

ST3.R24.C1****PHASE 1 DATA REDUCTION

DIGITS RELOW ARF DR IN EXCESS OF 16 DB. RANGE START IS 32143 SAMPLES. 96 SAMPLES PER RANGE

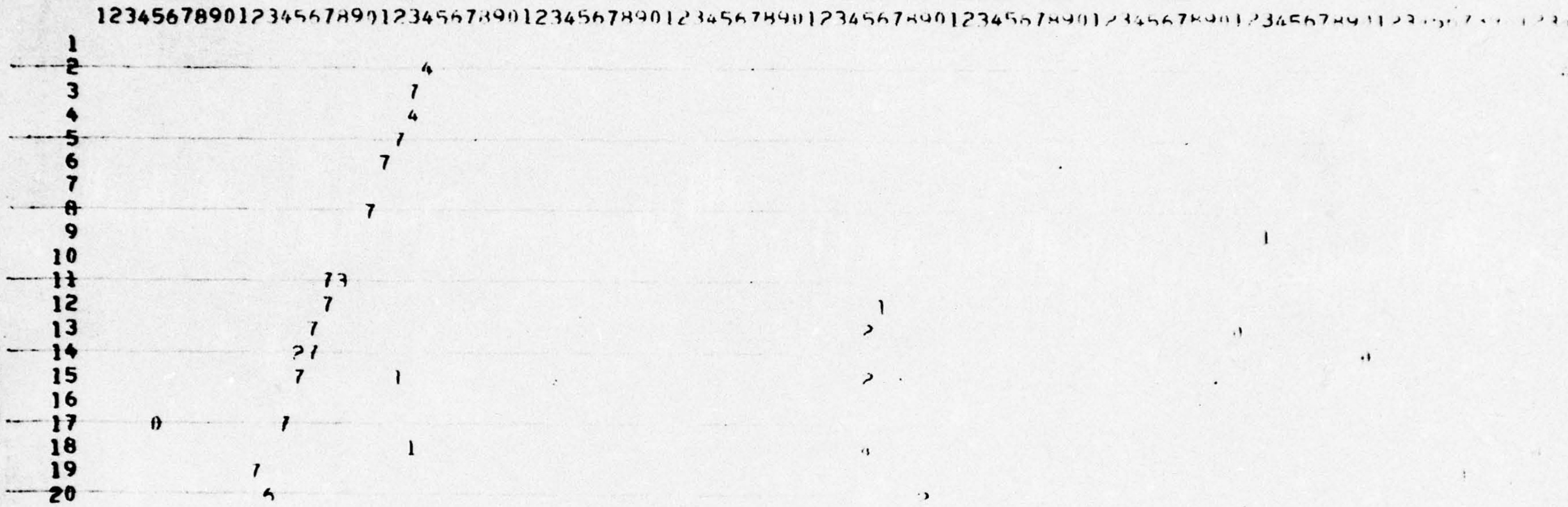

Fig. D-D5

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ST6.R3.C1****PHASE ONE DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF 11 DB. RANGE START IS 33137 SAMPLES. 103 SAMPLES PER RANGE BIN

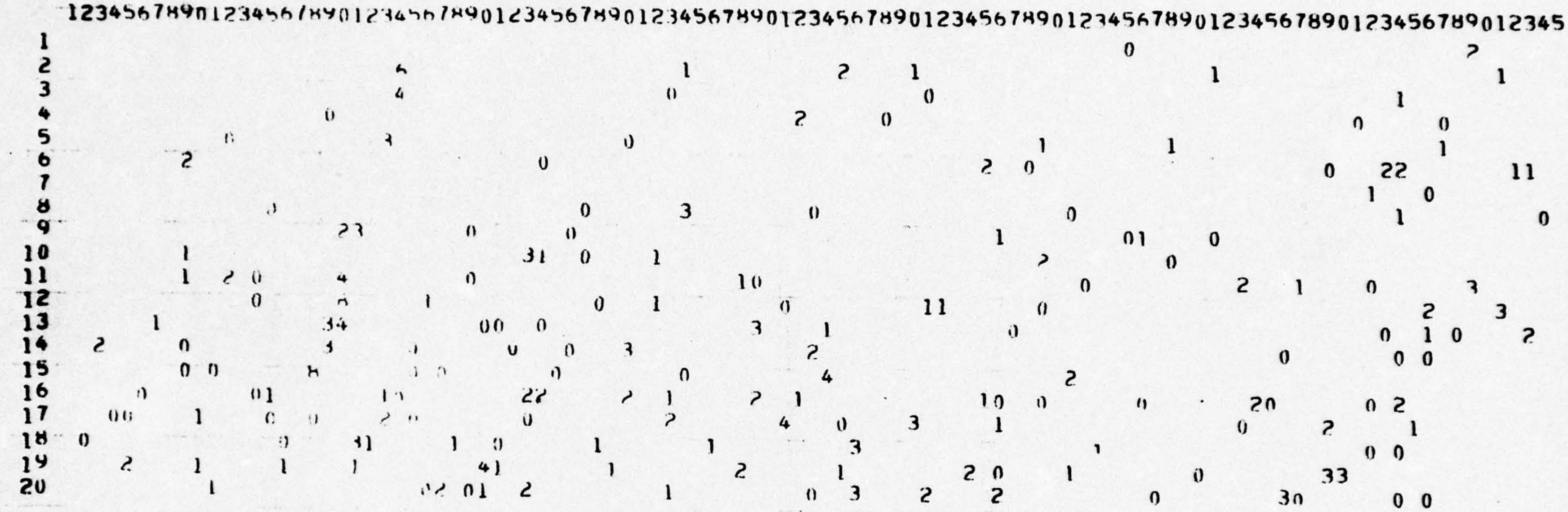

Fig. D-A6

PING NO.	SEC.	(S/N)	DEV
1			
2	35.[illegible]7[illegible]	16.59	18.6
3	35.[illegible]6[illegible]	14.73	-6.0
4			
5	35.484	14.40	-20.0
6			
7			
8			
9	35.283	13.72	-9.8
10			
11	35.158	15.27	14.3
12	35.101	19.04	19.5
13	35.105	15.04	-37.2
14	35.005	14.40	9.2
15	34.941	18.79	18.6
16			
17			
18	34.780	11.13	10.5
19	34.750	12.36	-17.6
20			

SIGMA = 18.34[illegible]

Fig. D-B6

```
                    ST6•R3•C1****PHASE ONE DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB. RANGE START IS 33137 SAMPLES.  103 SAMPLES PER RANGE BIN

    123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345
  1
  2                        6
  3                        4
  4
  5                       3
  6
  7
  8
  9                     3
 10
 11                    4
 12                    8
 13                    4
 14                   3
 15                  8
 16
 17
 18                0
 19                1
 20
```

Fig. D-C6

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```
                 ST6.63.C1****PHASE ONE DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF 16 DB. RANGE START IS 33137 SAMPLES.  103 SAMPLES PER RANGE BIN

    12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345
   1
   2                           1
   3                           3
   4
   5
   6
   7
   8
   9                        1
  10
  11                       3
  12                       7
  13                      7
  14                      7
  15                    7
  16
  17
  18                  5                                               0
  19                  7
  20                                   0                              1
```


Fig. D-D6

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```
                  ST6,R60,C1****PHASE ONE DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB, RANGE START IS 32385 SAMPLES,  105 SAMPLES PER RANGE BIN
    123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345
  1                       04        0
  2                       6          2                            1                    0
  3.        0           4             3        2                            1
  4                      9                                              0         2            0
  5                     05   0        2                                   0    1                        0
  6             0   0   2  0          1                                                    1
  7          1          3        1      0         2                                    0             1
  8                     1                                       1                00                 0
  9                   0                  0                2                 1      0      1            0
 10                   0      1       0                                1         0                  41
 11                  5                                     0                    0
 12                 70                                                0       0                0
 13      3          4      1      2   0       1         0   100       0        2    0   1
 14                7       2               2               4       3                   2       0
 15                6 0               0       00       4                 0 2      1                0
 16               5          0                  1   2    0     0                    0
 17               4    0       0  1             3           3    2            2    04     0    1 0
 18                             0                       0        0       1
 19  1       0   0          0         3        1                     2       02
 20             7    0         0          0           2             10           0     0       0
```

Fig. D-A7

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RING NO.	SEC.	(S/N)	DEV
1	34.424	15.38	6.8
2	34.746	17.43	25.6
3	34.690	19.67	22.4
4	34.659	20.16	-5.7
5	34.619	16.47	-24.7
6	34.560	13.48	-14.7
7	34.503	12.82	-26.6
8	34.447	11.91	-29.5
9	34.368	10.86	-9.4
10			
11	34.264	16.05	-22.8
12	34.163	17.89	14.5
13	34.091	21.26	32.9
14	34.039	18.23	26.3
15	33.984	16.75	22.8
16	33.929	16.42	19.4
17	33.979	15.33	11.0
18	33.831	10.07	.6
19	33.795	10.65	-21.7
20	33.742	17.49	-26.9

SIGMA = 21.[illegible]3

Fig. D-B7

COPY AVAILABLE TO DDC DOES NOT PERMIT FULLY LEGIBLE PRODUCTION

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COPY AVAILABLE TO DDC DOES NOT PERMIT FULLY LEGIBLE PRODUCTION

```
                    STG.R60.C1****PHASE ONE DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB. RANGE START IS 32385 SAMPLES.  105 SAMPLES PER RANGE BIN

       123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345
  1                           4
  2                          5
  3                         0
  4                         4
  5                         6
  6                        2
  7                        2
  8                       1
  9                      0
 10
 11                     5
 12                    7
 13                    9
 14                   7
 15                   6
 16                  
 17                  4
 18
 19                 0
 20                7
```

Fig. D-C7

CONFIDENTIAL

```
                              ST6.260.C1****PHASE ONE DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  16 DB. RANGE START IS 32385 SAMPLES.  105 SAMPLES PER RANGE BIN

    1234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345
  1
  2                         4
  3                        7
  4                        7
  5                        7
  6                       7
  7                       7
  8                      7
  9                     7
 10
 11                    7
 12                   7
 13                   7
 14                  7
 15                  7
 16                 7
 17                 7
 18                7
 19                7
 20               7
```

Fig. D-D7

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```
SI3.R29.C1****PHASE I DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB. RANGE START IS 34154 SAMPLES.   91 SAMPLES PER RANGE  10

   1234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
 1                    9                        2          00    1     1                   4                      0       3
 2     1         1     9                      2                                                                          3
 3                  0  5                                                                                    1    1
 4                   4 7                         0       0                                                  2  4
 5    01               9                                   3
 6                    8     2     1                 0              3           1     1                 13
 7                2   2    0     2                   5                                                         1
 8       53           9     1               1    1                           3                                       3
 9                    0
10                                       2            1
11   1                3      1    1                      5                10
12    0                        0   1                     5                1         1
13     2                              1                        3                    1       1        2           34
14   1  1              2       0              0                    0      2
15   1 0   2        13  3   0               1         2
16                        2         2           01        11            1
17    2              5          11          0                    1                                           1
18        3          0                    2      0     14            2                                        1
19                   4           9                     2
20                   2                            2              1    10                           3              2
```

Fig. D-A8

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PING NO.	SEC.	(S/N)	DEV
1	35.580	22.36	.7
2	35.569	21.46	-4.7
3	35.552	16.41	-4.2
4	35.629	17.52	2.2
5	35.611	21.24	3.3
6	35.597	19.91	.4
7	35.573	13.11	7.2
8	35.562	20.12	.9
9	35.542	10.56	3.4
10			
11	35.513	14.16	-3.1
12			
13	35.4[illegible]1	9.34	-7.[illegible]
14			
15	35.436	14.12	1.0
16			
17	35.413	16.10	-13.4
18	35.371	16.97	9.7
19	35.357	15.39	4.5
20	35.343	12.65	-0.[illegible]

SIGMA = 6.4[illegible]

Fig. D-B8

```
                    ST3.R29.C1****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB. RANGE START IS 34154 SAMPLES.   41 SAMPLES PER RANGE

     1234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
  1                      9
  2                      9
  3                      5
  4                      7
  5                      9
  6                     5
  7                     2
  8                     4
  9                     0
 10
 11                   3
 12
 13
 14
 15                   3
 16
 17                  5
 18                  0
 19                  4
 20                  2
```

Fig. D-C8

ST3.R23.C1****PHASE I DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF 16 DB. RANGE START IS 34154 SAMPLES. 91 SAMPLES PER RANGE [illegible]

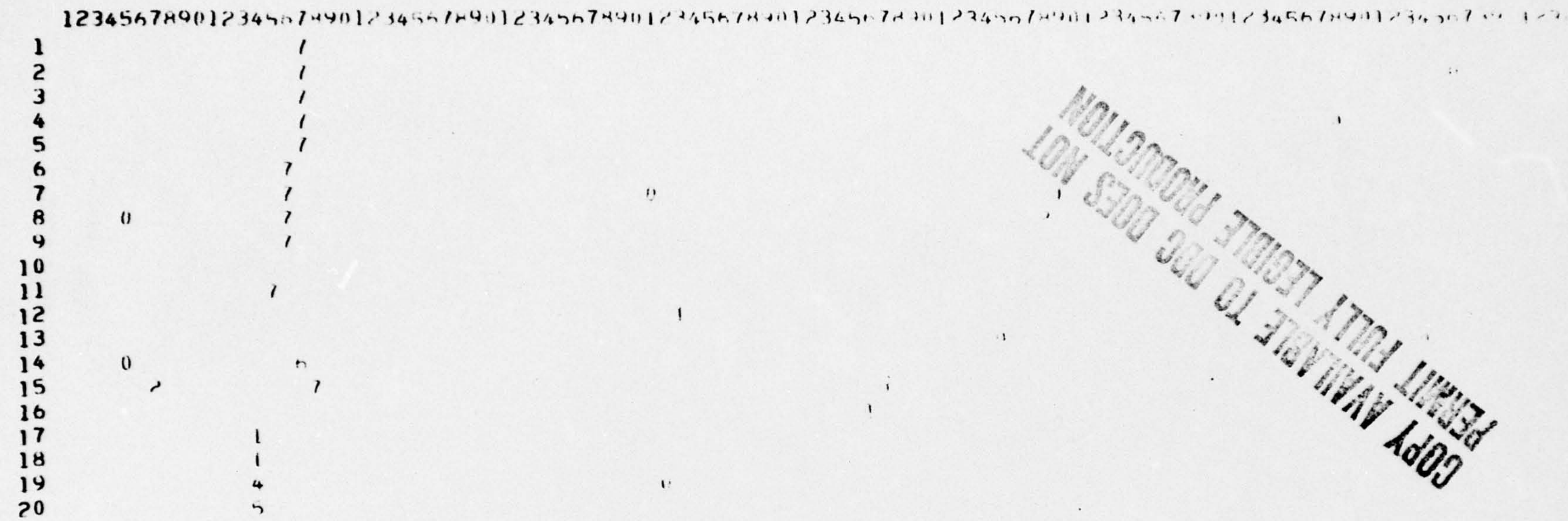

Fig. D-D8

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```
                 ST3.R2.C2****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB. RANGE START IS 34730 SAMPLES.  106 SAMPLES PER RANGE BIN

   12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
 1   0         1  8   0      0                       1                     0
 2     4          9                             0                       0              3            3
 3 7        0     9           1    1  5          2               0             1    1    0               3
 4                 9            2     8       1    1     3       0   0                1
 5       2         9                  6                 2                     0               0         2
 6        0        5    1     0        5    0     0              1             0 3
 7     0           0       0                                         0                             2
 8                  8        0                     0                                                      1
 9                  2               2    3      0                                     0          0
10                  1  1       1                               5                                 0    0
11     2    2       4     1                                                         0   1  2
12                  4                                                 0    0         1
13   01             9             1                    1  2                 1              0         3
14          3       6      0  0        3                  0 2                1     1    2              0 0
15           2      9   1  0    0      0  02   0           0     5      3             0                  1
16      1 0          70  33   12         3                        0              3               0      0
17    1              61              0       1                     3       1    0
18    8  3            5 10       0  1       1                                    0                   5    4
19    5         01    5                    5        0   0                   0   3  3     0
20     1   2          9                    1          2     0           0                1    4    0       3
```

Fig. D-A9

CONFIDENTIAL

PING NO.	SEC.	(S/N)	DEV
1	36.389	18.63	-3.8
2	36.411	21.39	-5.7
3	36.410	19.93	15.6
4	36.438	23.05	8.1
5	36.467	21.03	-0.3
6	36.493	15.87	-5.4
7	36.511	11.13	-2.4
8	36.542	18.66	-12.2
9	36.570	13.08	-18.8
10	36.558	12.47	14.8
11	36.595	15.04	-0.5
12	36.614	15.11	-2.5
13	36.620	22.71	12.6
14	36.653	17.35	2.9
15	36.672	19.55	11.4
16	36.711	18.29	-4.9
17	36.727	17.07	2.0
18	36.761	16.60	-9.0
19	36.757	15.93	-11.7
20	36.783	21.36	9.7

SIGMA = 9.370

Fig. D-B9

```
                         ST3.R2.C2****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB, RANGE START IS 34730 SAMPLES,  106 SAMPLES PER RANGE BIN

    12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
  1                8
  2                9
  3                9
  4                 4
  5                 9
  6                 5
  7                 0
  8                  5
  9                  2
 10                  1
 11                  4
 12                  4
 13                  4
 14                   6
 15                   9
 16                   7
 17                   0
 18                    5
 19                    7
 20                    4
```


Fig. D-C9

```
                    ST3.R2.C2****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  16 DB. RANGE START IS 34730 SAMPLES.  106 SAMPLES PER RANGE BIN

   12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
 1                3
 2                7
 3 2              7                   0
 4                 7                  5
 5                 7                  7
 6                 7                   7
 7                 7
 8                  7
 9                  7                    7
10                  7                                          0
11                  7
12                  7
13                  7
14                   7
15                   7                      3                     0
16                   7                   1                        1
17                   7                       2                     1
18    3               7                                                                              0
19    6               7                    2
20     0              7
```

Fig. D-D9

```
                      ST3.R7.C1****PHASE 1 DATA REDUCTION

IGITS BELOW ARE DB IN EXCESS OF  11 DB, RANGE START IS 34577 SAMPLES,   98 SAMPLES PER RANGE BIN

    12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
  1                     1     9                       0           1                  2             0    0
  2                      0     0                     2             1    4
  3             7           21  2           1   0        4  11     0   1   0       2   0            0
  4            40        1               2      2                    0      1
  5         0  0    2       0   5                            1   2                        3                40
  6                         0   12                                                             0 0       0
  7                      0          4          3     1  04           0                0                03
  8           2                  2      1                                              0
  9           6           2      7             0  0        1           0      2        1 2     0    1  1  1
 10          6   2  0  1  4    0             0      2            10                        0         0
 11          72       2   5     2               30            0                       2      1       1
 12     3    9      1  0    0   0            3        4          2                  3  2     1 2
 13          9          4                      5       0      2   1                      0       22
 14         9     3        2    1      2          1   0  0     5         0 0                       0    1
 15         7           3 1 2    2          6              0  0                              0         2   5
 16         67   0  1         0      1           2 2  0       2      0     0    1      1 30          0
 17        1        1        51     0      0                           3  1   0  1
 18     1    7      1        16     0              0      1        1                     2
 19        3     2    2           0   1                     4 2   12      1             1 02            10
 20      1      0     103          0          1       6   0      2    0                      0    2      0
```

Fig. D-A10

PING NO.	SEC.	(S/N)	DEV
1			
2	35.736	10.01	-10.0
3	35.672	18.24	15.6
4	35.643	15.26	6.0
5	35.620	11.06	-9.9
6			
7			
8	35.507	13.28	-14.5
9	35.461	17.22	-8.2
10	35.399	17.37	14.0
11	35.350	18.18	22.9
12	35.332	20.62	.7
13	35.298	21.72	-5.7
14	35.259	20.74	-7.4
15	35.218	18.08	-7.2
16	35.178	16.69	-8.3
17			
18	35.070	18.40	17.0
19	35.045	13.72	-0.6
20	35.008	10.26	-4.5

SIGMA = 11.157

Fig. D-B10

```
                 ST3.R7.C1***PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB, RANGE START IS 34577 SAMPLES.   98 SAMPLES PER RANGE BIN

    12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
   1
   2
   3          7
   4         4
   5         0
   6
   7
   8        2
   9        6
  10       6
  11      7
  12      4
  13      9
  14     9
  15     7
  16     6
  17
  18    7
  19   3
  20
```

Fig. D-C10

```
                 SI3,R7,C1****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  16 DB, RANGE START IS 34577 SAMPLES,   98 SAMPLES PER RANGE BIN

    12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
  1                           4
  2                            4
  3            2
  4           4
  5           5                0
  6                            1
  7                             4
  8          4                  5
  9          7                  7
 10         7            0
 11        72            4     7
 12        7              7   7
 13        7                                  0
 14       7              7                                   1
 15       7               7               1                                                             0
 16      77
 17                      74
 18      7               7
 19     7
 20     7           1                             0
```

Fig. D-D10

CONFIDENTIAL

ST3.R1.C1****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF 11 DB, RANGE START IS 33911 SAMPLES. 110 SAMPLES PER RANGE BIN

```
     123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
   1                3                                                        1            0                   4
   2                1                         1   0                                                 0
   3               00     0      2    0     1                 0                  1                       0          2
   4        2       9                                                                           0
   5            1    9   1                                                    0     0                0               1
   6       1         9                                                  0                                             2
   7           3     9       4               0                 1                    0
   8   0             1           1         1         0                0              1                           1
   9                                                                0    2         1                        1
  10                 5      0        01              2          3   0                3            1                  0
  11                 9                     0    0   0                            1 1                   0           3
  12                  7   0                                                                         1 0           0   2
  13     2  2         9     0                    2    3  0      2                          02          1       12    0
  14    1             4           3               2      0                        2                       1          0
  15               0        01       0    1     0  1                              0                           1
  16       0             0             1           1 0                1 5                                         3
  17   4           3                                                       2                    1        3    0
  18 0                      2                        1  0
  19    1                   1    05                1 0         2               0 2
  20   1                  0       3            0                                                              5
```

Fig. D-A11

PING NO.	SEC.	(S/N)	DEV
1	35.406	14.30	-4.4
2	35.417	11.89	.3
3	35.438	11.05	-5.0
4	35.430	23.45	18.6
5	35.467	20.98	-2.8
6	35.480	19.51	-0.3
7	35.499	19.68	-3.7
8	35.513	12.26	-2.2
9			
10	35.547	16.12	-5.3
11	35.557	22.92	.1
12	35.567	17.95	5.5
13	35.586	21.90	1.9
14	35.606	14.94	-2.8
15	35.619	9.69	-0.4
16			
17			
18			
19	35.679	9.69	.6
20			
SIGMA =	5.698		

Fig. D-B11

```
                    ST3.R1.C1****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  16 DB, RANGE START IS 33911 SAMPLES.  110 SAMPLES PER RANGE BIN

     123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
  1
  2
  3                0
  4                7
  5                 7
  6                 7
  7                 7
  8                 7
  9
 10                 7
 11                 7
 12                  7
 13                  7
 14                  7
 15                  7
 16                                                                     0
 17
 18
 19                              0
 20                              2                                                                            0
```


Fig. D-D11

```
                     ST3,R1,C1****PHASE 1 DATA REDUCTION

DIGITS BELOW ARE DB IN EXCESS OF  11 DB, RANGE START IS 33911 SAMPLES,  110 SAMPLES PER RANGF BIN

     1234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234
   1             3
   2             1
   3             0
   4             9
   5              9
   6              9
   7              9
   8              1
   9
  10              5
  11              9
  12               7
  13               9
  14               4
  15
  16
  17
  18
  19
  20
```

Fig. D-C11